

Poznámky k 5. přednášce



$$\underline{y}''(t) + a_1 \underline{y}'(t) + a_0 \underline{y}(t) = u(t)$$

$$y(0) = c_1 \quad ; \quad y'(0) = c_2$$

a) Rad systému = nejvyšší derivace
= 2

$$x_1(t) = y(t) \leftarrow \text{wystep}$$

$$x_2(t) = \underline{y'(t)}$$

$$b) \quad x_1'(t) = \underline{y'(t)} = x_2(t)$$

$$x_2'(t) = y''(t) = -a_0 \cdot x_1(t) - a_1 \cdot x_2(t) + u(t) \quad \left\{ \begin{array}{l} y''(t) + a_1 y'(t) + a_0 y(t) = u(t) \\ y''(t) = -a_0 \cdot y(t) - a_1 y'(t) + u(t) \end{array} \right.$$

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$g(0) \rightarrow x_1(0) = c_1$$

$$y'(0) \rightarrow x_2(0) = c_2$$

$D=0$ i tzu. rygi system

Obecně:

$$y^{(n)}(t) + a_{n-1}y^{(n-1)}(t) + \cdots + a_1y^{(1)}(t) + a_0y(t) = u(t)$$

vid systemu je w

$$x_1(t) = y(t), \quad x'_1(t) = \underline{y'(t)} \quad x'_1(t) = x_2(t),$$

$$x_2(t) = y'(t), \quad x_2'(t) = y''(t)$$

$$x_3(t) = y''(t),$$

$$x_4(t) = y^{(3)}(t), \quad x'_{n-1}(t) = x_n(t),$$

$$\vdots \qquad \qquad x'_n(t) = u(t) - a_0x_1(t) - a_1x_2(t) - \dots - a_{n-1}x_n(t).$$

$$x_n(t) = y^{(n-1)}(t),$$

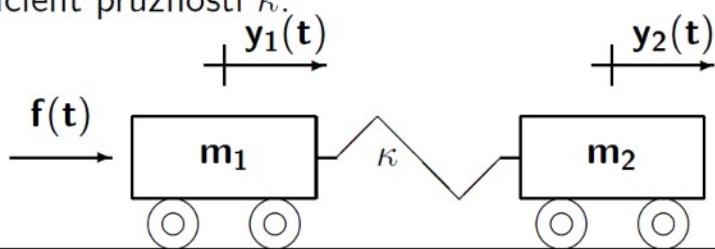
maticový zápis

$$\begin{bmatrix} x'_1(t) \\ x'_2(t) \\ x'_3(t) \\ \vdots \\ x'_n(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & & \\ -a_0 - a_1 - a_2 - a_3 - \cdots - a_{n-1} & & & & & \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u(t)$$

$$y = [1 \ 0 \ 0 \ 0 \ \dots \ 0] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix} \quad \mathfrak{D} = 0$$

Príklad:

Dva vozíky s hmotností m_1 a m_2 jsou spojeny pružinou, která má koeficient pružnosti κ .



$$m_1 \cdot y_1''(t) = f(t) + k(y_2(t) - y_1(t)) \rightarrow y_1''(t) = \frac{1}{m_1} f(t) + \frac{k}{m_1} y_2(t) - \frac{k}{m_1} y_1(t)$$

$$m_2 \cdot y_2''(t) = -k(y_2(t) - y_1(t)) \rightarrow y_2''(t) = -\frac{k}{m_2} y_2(t) + \frac{k}{m_2} y_1(t)$$

řád systému: $2 + 2 = 4$

a) $x_1(t) = y_1(t)$ b) $x_1'(t) = y_1'(t) = x_2(t)$
 $x_2(t) = y_1'(t)$ $x_2'(t) = y_1''(t) = -\frac{k}{m_1} x_1(t) + \frac{k}{m_1} x_3(t) + \frac{1}{m_1} f(t)$
 $x_3(t) = y_2(t)$ $x_3'(t) = y_2'(t) = x_4(t)$
 $x_4(t) = y_2'(t)$ $x_4'(t) = y_2''(t) = \frac{k}{m_2} x_1(t) - \frac{k}{m_2} x_3(t)$

$$\begin{bmatrix} x'_1(t) \\ x'_2(t) \\ x'_3(t) \\ x'_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & 0 & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} f(t)$$

$$\begin{bmatrix} x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ \frac{\kappa}{m_2} & 0 & -\frac{\kappa}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

$$x_1'(t) = y_1'(t) = x_3(t)$$

stage:

$$x_1(t) = y_1(t), \quad x_2(t) = y_2(t),$$

$$x_3(t) = y_1'(t), \quad x_4(t) = y_2'(t)$$

derivative



$$x_1'(t) \equiv y_1'(t) = x_3(t),$$

$$x_2'(t) \equiv y_2'(t) = x_4(t),$$

$$x_3'(t) \equiv y_1''(t) = \frac{\kappa}{m_1} (x_2(t) - x_1(t)) + \frac{1}{m_1} f(t),$$

$$x_4'(t) \equiv y_2''(t) = -\frac{\kappa}{m_2} (x_2(t) - x_1(t)).$$

$$\mathbf{A} \begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \\ x_4'(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\kappa}{m_1} & \frac{\kappa}{m_1} & 0 & 0 \\ \frac{\kappa}{m_2} & -\frac{\kappa}{m_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} f(t)$$

diskrétní systém



$$y[n+2] + \alpha_1 y[n+1] + \alpha_0 y[n] = u[n]$$

$$y[0] = c_1 \quad y[1] = c_2$$

$$x[n+1] = A \cdot x[n] + B \cdot u[n]$$

$$y[n] = C \cdot x[n] + D \cdot u[n]$$

čidlo systému: nejvýšší posunutí v čase
: 2

a) $x_1[n] = y[n]$ / výstup
 $x_2[n] = y[n+1]$

b) $x_1[n+1] = y[n+1] = x_2[n]$

$$x_2[n+1] = y[n+2] = -\alpha_0 \cdot x_1[n] - \alpha_1 \cdot x_2[n] + u[n]$$

$$y[n+2] = -\alpha_0 \cdot y[n] - \alpha_1 \cdot y[n+1] + u[n]$$

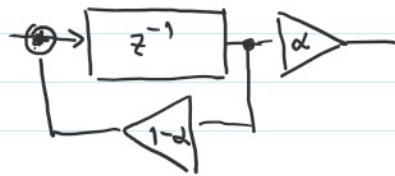
↑ ↑
 $x_1[n]$ $x_2[n]$

$$\begin{matrix} \uparrow \\ x_1[n] \end{matrix} \quad \begin{matrix} \uparrow \\ x_2[n] \end{matrix}$$

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha_0 & -\alpha_1 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[n]$$

$$y[n] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} \quad D = 0$$

$$\begin{aligned} y(0) &\approx x_1(0) = c_1 \\ y(1) &\approx x_2(0) = c_2 \end{aligned}$$



Vnigjstí popis ← vnitřní popis

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\begin{aligned} y(t) &= x_1(t) \\ y'(t) &= x_2(t) \end{aligned} \xrightarrow{\frac{d}{dt}} y''(t) = x_2'(t) = 2 \cdot x_1(t) + 3 \cdot x_2(t) + u(t)$$

$$y''(t) = 2 \cdot y(t) + 3 \cdot y'(t) + u(t)$$

$$y''(t) - 3 \cdot y'(t) - 2 \cdot y(t) = u(t)$$