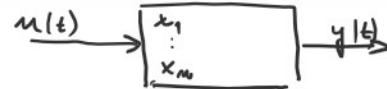
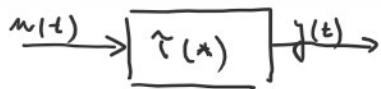


## Poznámky k 6. přednášce



$$y''(t) + \alpha \cdot y'(t) + \beta \cdot y(t) = u(t)$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = A \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + B \cdot u(t)$$

$$y(t) = C \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + D \cdot u(t)$$

$$y^{(4)}(t) + \alpha \cdot y''(t) + \beta \cdot \int y(\tau) d\tau + \underbrace{\int f(t-\tau) g(\tau) d\tau}_{\text{konvoluce}} = u(t)$$

$\uparrow p^4 \dots \quad p^3 \dots \quad \frac{1}{p} \dots \dots \dots F(p) \cdot G(p)$

$$f(t) = a_0 + a_1 t + a_2 t^2 + \dots + a_m t^m$$

$$f(t) \xrightarrow{\mathcal{L}} F(p) \quad ; \quad F(p) = \mathcal{L}\{f(t)\}$$

!

$$F(p) = \int_0^\infty f(t) e^{-pt} dt$$

obraz      vzor

!

$$\mathcal{L}\{\alpha \cdot f(t) \pm \beta \cdot g(t)\} = \mathcal{L}\{\alpha \cdot f(t)\} \pm \mathcal{L}\{\beta \cdot g(t)\} = \alpha \mathcal{L}\{f(t)\} \pm \beta \mathcal{L}\{g(t)\}$$

Linearity

!

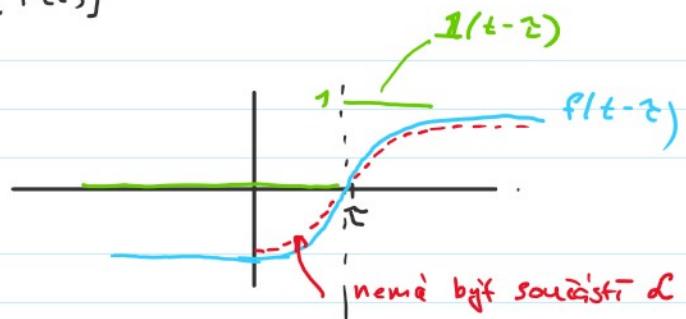
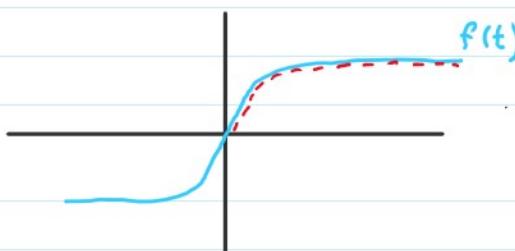
$$\mathcal{L}\{f(a \cdot t)\} = \frac{1}{a} F\left(\frac{p}{a}\right) \quad \text{změna měřítka}$$

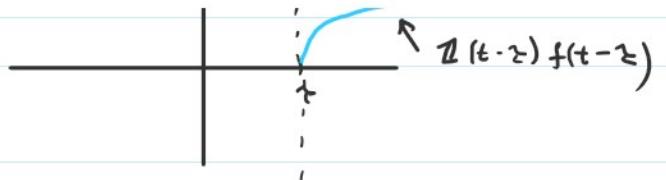
Posunutí

!

$$F(p) = \mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{\underbrace{L(t-\tau)}_{\text{impulse}} f(t-\tau)\} = e^{-p\tau} \cdot \mathcal{L}\{f(t)\}$$





O konvoluci

$$! \quad \mathcal{L} \left\{ f(t) * g(t) \right\} = \mathcal{L} \left\{ \int_0^{\infty} f(t-z) \cdot g(z) dz \right\} = F(p) \cdot G(p)$$

$$\mathcal{L} \left\{ y(t) \right\} = \int_0^{\infty} h(z) \cdot u(t-z) dz \Rightarrow Y(p) = H(p) \cdot U(p)$$

výstup  
 ↑  
 impulsní  
 ↓  
 odesra  
 vstup  
 ↑  
 výstup  
 ↓  
 přenosová  
 funkce  
 ↑  
 vstup

O obrazu elenace

$$\mathcal{L} \left\{ f(t) \right\} \rightarrow F(p)$$

$$\mathcal{L} \left\{ f'(t) \right\} \rightarrow pF(p) - f(0)$$

$$\mathcal{L} \left\{ f''(t) \right\} \rightarrow p^2 F(p) - p \cdot f(0) - f'(0)$$

⋮

$$\mathcal{L} \left\{ \frac{d^n}{dt^n} f(t) \right\} = p^n F(p) - p^{n-1} f(0) - p^{n-2} \frac{d}{dt} f(0) \dots - f^{(n-1)}(0)$$

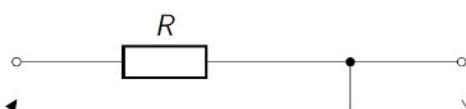
O obrazu integrálů

$$! \quad \mathcal{L} \left\{ \int_0^t f(z) dz \right\} = \frac{1}{p} \cdot F(p)$$

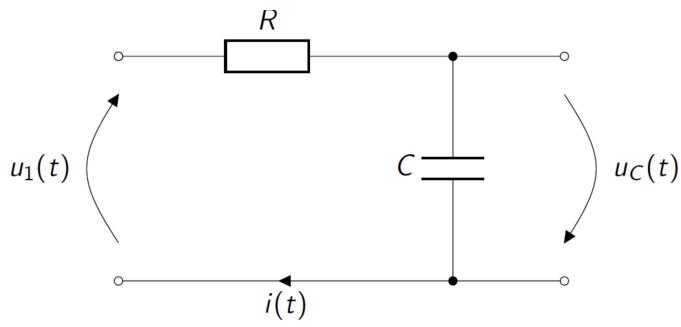
$$\begin{aligned} \mathcal{L} \left\{ \frac{\sin t}{f(t)} \right\} &= \int_0^{\infty} (\sin t) \cdot e^{-pt} dt = \\ &= \frac{1}{p^2 + 1} \end{aligned}$$

Příklad

odesra integračního RC čidulek



$f(t) = \mathcal{L}^{-1}\{F(p)\}$	$F(p) = \mathcal{L}\{f(t)\}$
$\delta(t)$	1
$\mathbf{1}(t)$	$\frac{1}{p}$
$e^{-\alpha t}$	$\frac{1}{p + \alpha}$
$\sin \omega t$	$\frac{\omega}{p^2 + \omega^2}$
$\cos \omega t$	$\frac{p}{p^2 + \omega^2}$
$e^{-\alpha t} \sin \omega t$	$\frac{\omega}{(p + \alpha)^2 + \omega^2}$
$e^{-\alpha t} \cos \omega t$	$\frac{p + \alpha}{(p + \alpha)^2 + \omega^2}$
$t^n$	$\frac{n!}{p^{n+1}}$



$$RC \cdot u'_c(t) + u_c(t) = u_1(t)$$

$$\left\{ \begin{array}{l} \omega = \frac{1}{RC} \quad ; \quad u_c(t) \approx y(t) \\ u_1(t) = U_0 \end{array} \right\}$$

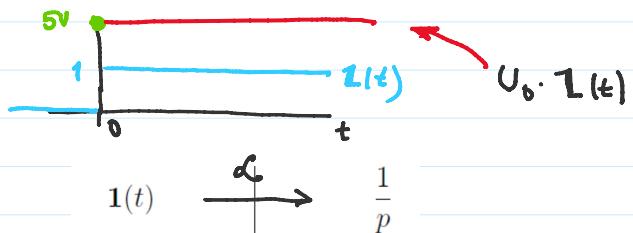
$$y'(t) + \alpha \cdot y(t) = \alpha \cdot U_0 \cdot \mathbb{1}(t)$$

$$p \cdot Y(p) - y(0) + \alpha \cdot Y(p) = \alpha \cdot U_0 \cdot \frac{1}{p}$$

$$(p + \alpha) Y(p) = \frac{\alpha \cdot U_0}{p} + y(0)$$

$$Y(p) = \frac{\alpha \cdot U_0}{p(p + \alpha)} + \frac{y(0)}{p + \alpha}$$

$t^n$	$\frac{n!}{p^{n+1}}$
$t^n e^{-\alpha t}$	$\frac{n!}{(p + \alpha)^{n+1}}$
$t \cos \omega t$	$\frac{p^2 - \omega^2}{(p^2 + \omega^2)^2}$
$t \sin \omega t$	$\frac{2\omega p}{(p^2 + \omega^2)^2}$
$\sinh \varphi t$	$\frac{\varphi}{p^2 - \varphi^2}$
$\cosh \varphi t$	$\frac{p}{p^2 - \varphi^2}$
$t \sinh \varphi t$	$\frac{2\varphi p}{(p^2 - \varphi^2)^2}$
$t \cosh \varphi t$	$\frac{p^2 + \varphi^2}{(p^2 - \varphi^2)^2}$



$$y(t) = \mathcal{L}^{-1}\{F(p)\}$$

Rozložení na parcíální zlomky

$$\frac{\alpha \cdot U_0}{p(p + \alpha)} = \alpha \cdot U_0 \cdot \frac{1}{p(p + \alpha)} = \alpha \cdot U_0 \left[ \frac{\frac{1}{\alpha}}{p} - \frac{\frac{1}{\alpha}}{p + \alpha} \right] = \frac{U_0}{p} - \frac{U_0}{p + \alpha}$$

$$\left\{ \frac{1}{p(p + \alpha)} = \frac{A}{p} + \frac{B}{p + \alpha} \right.$$

$$A = \lim_{p \rightarrow 0} \frac{1}{(p + \alpha)} = \frac{1}{\alpha} \quad ; \quad B = \lim_{p \rightarrow -\alpha} \frac{1}{p} = -\frac{1}{\alpha}$$

$$Y(p) = \frac{U_0}{p} - \frac{U_0}{p + \alpha} + \frac{y(0)}{p + \alpha}$$

$$y(t) = U_0 \cdot \mathbb{1}(t) - U_0 \cdot e^{-\alpha t} + y(0) \cdot e^{-\alpha t}$$

$$\alpha = \frac{1}{RC}$$

$$\frac{U_0}{p}$$

$$1(t) \xrightarrow{\mathcal{L}} \frac{1}{p}$$

$$e^{-\alpha t} \xrightarrow{\mathcal{L}} \frac{1}{p + \alpha}$$

$$e^{-\alpha t} \rightarrow \frac{1}{p + \alpha}$$

$$\alpha = \omega$$

$$\frac{U_0}{p+\alpha} e^{-\alpha t} \xrightarrow{\alpha = \omega} r + w$$

Uvažujte LTI systém, který je pro  $t > 0$  popsán naměřenými hodnotami vstupu

$$u(t) = e^{-t} + e^{-3t}$$

a výstupu

$$y(t) = te^{-3t}.$$



Jak nalezneme **impulsní odezvu**?

$$\xrightarrow{h(t)}$$

$$y(t) = \int_0^{\infty} h(\tau) \cdot u(t-\tau) d\tau \xrightarrow{\mathcal{L}} H(p) = U(p) \cdot V(p)$$

$$h(t) = \mathcal{L}^{-1}\{H(p)\}$$

$$H(p) = \frac{V(p)}{U(p)}$$

$$y(t) = t \cdot e^{-3t} \rightarrow V(p) = \frac{1}{(p+3)^2}$$

$$\frac{n!}{(p+\alpha)^{n+1}}$$

$$\left\{ \begin{array}{l} n=1 \\ \alpha=3 \end{array} \right.$$

$$u(t) = e^{-t} + e^{-3t} \rightarrow U(p) = \frac{1}{p+1} + \frac{1}{p+3}$$

$$e^{-\alpha t}$$

$$\frac{1}{p+\alpha}$$

$$H(p) = \frac{V(p)}{U(p)} = \frac{\frac{1}{(p+3)^2}}{\frac{1}{p+1} + \frac{1}{p+3}} = \frac{\frac{1}{(p+3)^2}}{\frac{p+3+p+1}{(p+1)(p+3)}} = \frac{1}{(p+3)^2} \cdot \frac{(p+1)(p+3)}{2p+4}$$

$$U(p) = \frac{p+1}{2(p+2)(p+3)}$$

$$h(t) = \mathcal{L}^{-1}\{H(p)\}$$

$$\frac{1}{2} \left[ \frac{p+1}{(p+2)(p+3)} \right] = \frac{1}{2} \left[ \frac{A}{p+2} + \frac{B}{p+3} \right] = \frac{1}{2} \cdot \left[ \frac{-1}{p+2} + \frac{2}{p+3} \right]$$

$$\left\{ \begin{array}{l} A = \lim_{p \rightarrow -2} \frac{p+1}{p+3} = \frac{-1}{1} = -1 \\ B = \lim_{p \rightarrow -3} \frac{p+1}{p+2} = \frac{-2}{-1} = 2 \end{array} \right.$$

$$B = \lim_{p \rightarrow -3} \frac{p+1}{p+2} = \frac{-2}{-1} = 2$$

$$B = \lim_{p \rightarrow -1} \frac{p+1}{p+2} = \frac{-2}{-1} = 2$$

$$H(p) = \frac{1}{p+3} - \frac{1}{2} \cdot \frac{1}{p+2} e^{-at}$$

$$h(t) = \mathcal{L}^{-1}\{H(p)\} = e^{-3t} - \frac{1}{2} \cdot e^{-2t}$$

$$\frac{1}{p+\alpha}$$

