$$R(p) = \frac{Q(p)}{N(p)} = \frac{b_m p^m + b_{m-1} p^{m-1} + \cdots + b_1 p + b_0}{a_n p^n + a_{n-1} p^{n-1} + \cdots + a_1 p + a_0} \qquad m < n$$

$$N(p) = \prod_{\mu=1}^{n} (p - p_{\infty\mu}) = (p - p_{\infty 1})(p - p_{\infty 2}) \ldots (p - p_{\infty n}).$$

$$\frac{Q(p)}{N(p)} = \sum_{\mu=1}^{n} \frac{k_\mu}{p - p_{\infty\mu}} \qquad k_1, k_2, \cdots, k_n \sim \text{residua}$$

$$= \frac{k_1}{p - p_{\infty 1}} + \frac{k_2}{p - p_{\infty 2}} + \cdots + \frac{k_n}{p - p_{\infty n}}$$

$$\equiv \frac{k_1}{p - p_1} + \frac{k_2}{p - p_2} + \cdots + \frac{k_n}{p - p_n}, \qquad p_1, p_2 \cdots p_n \text{ nulové body}$$

$$R(p) = \frac{Q(p)}{(p-p_1)^{m_1}(p-p_2)^{m_2}\cdots(p-p_N)^{m_N}(p^2+\alpha_1 p+\beta_1)^{m_1}\cdots(p^2+\alpha_M p+\beta_M)^{m_M}}$$

$(p-1)(p-1) \sim (p-1)^2$

reálné kořeny — komplexní kořeny

I. Nenásobné reálné kořeny

a) $R(p) = \frac{p+5}{p^2-2p-3} = \frac{p+5}{(p-3)(p+1)} = \frac{A}{p-3} + \frac{B}{p+1}$

a) $\frac{p+5}{(p-3)(p+1)} = \frac{A}{p-3} + \frac{B}{p+1} = \frac{A(p+1)+B(p-3)}{(p-3)(p+1)} = \frac{Ap+A+Bp-3B}{(p-3)(p+1)}$

$5 \sim 5 \cdot p^0$ $\quad 1 \cdot p + 5 = (A+B)p + (A-3B)$ $\quad a_2 p^2 + a_1 \cdot p + a_0 = b_2 \cdot p^2 + b_1 p + b_0$

$1 = A + B$

$5 = A - 3B \quad / -$

$-4 = 4B \Rightarrow B = -1 \,;\; A = 2$

b) $R(p) = \frac{p+5}{(p-3)(p+1)} = \frac{A}{p-3} + \frac{B}{p+1} = \frac{2}{p-3} - \frac{1}{p+1}$

$A = \lim_{p \to 3} \frac{p+5}{(////)(p+1)} = \frac{8}{4} = 2$ $\qquad B = \frac{p+5}{(p-3)(////)}\Big|_{p=-1} = \frac{4}{-4} = -1$

$$A = \lim_{p \to 3} \frac{p+5}{(p-3)(p+1)} = \frac{8}{4} = 2 \qquad B = \frac{p+5}{(p-3)(p+1)}\Big|_{p=-1} = \frac{4}{-4} = -1$$

II. Násobné reálné kořeny

$$\frac{}{p^2(p-1)} = \frac{}{p \cdot p\,(p-1)}$$

a) $$R(p) = \frac{2p^2-1}{p^3-p^2} = \frac{2p^2-1}{p^2(p-1)} = \frac{A}{p} + \frac{B}{p^2} + \frac{C}{p-1}$$

(A: ?; B, C: limitní metoda)

$$C = \frac{2p^2-1}{p^2}\Big|_{p=1} = \frac{1}{1} = 1$$

$$B = \frac{2p^2-1}{\not{p}(p-1)}\Big|_{p=0} = \frac{-1}{-1} = 1$$

$$\frac{}{p^3(p-1)^2} = \frac{A}{p} + \frac{B}{p^2} + \frac{C}{p^3} + \frac{D}{p-1} + \frac{E}{(p-1)^2}$$

$$\frac{2p^2-1}{p^2(p-1)} = \frac{A}{p} + \frac{1}{p^2} + \frac{1}{p-1}$$

i) $$\frac{2p^2-1}{p^2(p-1)} = \frac{Ap(p-1)+(p-1)+p^2}{p^2(p-1)} = \frac{(A+1)p^2+(1-A)p-1}{p^2(p-1)}$$

$$2p^2-1 = (A+1)p^2 + (1-A)p - 1$$

$$2 = A+1 \Rightarrow \underline{A=1}$$

ii) $$2p^2-1 = Ap(p-1) + (p-1) + p^2$$ ($A \cdot p^2 - A$)

$\frac{2p^2-1}{p^2(p-1)}$ — $p=1$, $p=0$

$p=-1$: $1 = 2A - 2 + 1$

$$2A = 2 \Rightarrow \underline{A=1}$$

iii) $$\frac{2p^2-1}{p^2(p-1)} = \frac{A}{p} + \frac{B}{p^2} + \frac{C}{p-1} \quad /\,p$$

$$\frac{2p^3-p}{p^3-p^2} = A + \frac{B}{p} + \frac{Cp}{p-1}$$

$\lim_{p\to\infty} \frac{p \cdot C}{p(1-\frac{1}{p})} = C$

$C=1$, $B=1$

lim $p \to \infty$

$$\lim_{p\to\infty} \frac{2p^3-p}{p^3-p^2} = \lim_{p\to\infty} \frac{p^3(2-\frac{1}{p^2})}{p^3(1-\frac{1}{p})} = 2$$

$$2 = A + 0 + C$$

$$2 = A + 1 \Rightarrow \underline{A=1}$$

b) $$R(p) = \frac{1}{} = \frac{A}{} + \frac{B}{} + \frac{C}{}$$

b)

$$R(p) = \frac{1}{(p+1)^2(p+2)} = \frac{A}{p+1} + \frac{B}{(p+1)^2} + \frac{C}{p+2}$$

$$\frac{1}{(p+1)^2(p+2)} = \frac{1}{p+1}\left[\frac{1}{(p+1)(p+2)}\right] = \frac{1}{p+1}\left[\frac{A}{p+1} + \frac{B}{p+2}\right] =$$

$$= \left\{ A = \frac{1}{(p+1)(p+2)}\Big|_{p=-1} = \frac{1}{1} = 1 \right.$$
$$\left. B = \frac{1}{(p+1)(p+2)}\Big|_{p=-2} \quad \frac{1}{-1} = -1 \right\} =$$

$$= \frac{1}{p+1}\left[\frac{1}{p+1} - \frac{1}{p+2}\right] = \frac{1}{(p+1)^2} - \frac{1}{(p+1)(p+2)} =$$

$$= \frac{1}{(p+1)^2} - \left[\frac{1}{(p+1)(p+2)}\right] = \frac{1}{(p+1)^2} - \left[\frac{A}{p+1} + \frac{B}{p+2}\right] =$$

$$= \frac{1}{(p+1)^2} - \frac{1}{p+1} + \frac{1}{p+2}$$

III. Komplexní kořeny

$$R(p) = \frac{Q(p)}{p(p^2+6p+10)} = \frac{A}{p} + \frac{Bp+C}{p^2+6p+10}$$

$$R(p) = \frac{2p+3}{p^2+6p+10}$$

$$p^2+6p+10 = (p+\alpha)^2 + \omega^2 = (p+3)^2 + 1$$

$$(p+3)^2 = p^2+6p+9 \quad \begin{cases} \alpha = 3 \\ \omega = 1 \end{cases}$$

$$\frac{2p+3}{p^2+6p+10} = \frac{2p+3}{(p+3)^2+1} =$$

$$= 2\frac{p+3}{(p+3)^2+1} - \frac{3}{(p+3)^2+1}$$

$2p+6$, -3

$$\mathcal{L}^{-1}\left\{\frac{2p+3}{\quad}\right\} =$$

$\mathbf{1}(t)$	$\frac{1}{p}$
t^n	$\frac{n!}{p^{n+1}}$
$e^{-\alpha t}$	$\frac{1}{p+\alpha}$
$\sin \omega t$	$\frac{\omega}{p^2+\omega^2}$
$\cos \omega t$	$\frac{p}{p^2+\omega^2}$
$e^{-\alpha t}\sin \omega t$	$\frac{\omega}{(p+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos \omega t$	$\frac{p+\alpha}{\quad}$

$(p+3)+1$ $\quad$ $(p+3)^2+1$

$$\mathcal{L}^{-1}\left\{\frac{2p+3}{p^2+6p+10}\right\} =$$

$e^{-\alpha t}\sin\omega t$	$\frac{\omega}{(p+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos\omega t$	$\frac{p+\alpha}{(p+\alpha)^2+\omega^2}$

$$2\mathcal{L}^{-1}\left\{\frac{p+3}{(p+3)^2+1}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{(p+3)^2+1}\right\} = \underbrace{2\cdot e^{-3t}\cdot\cos t - 3e^{-3t}\cdot\sin t}_{r(t)}$$