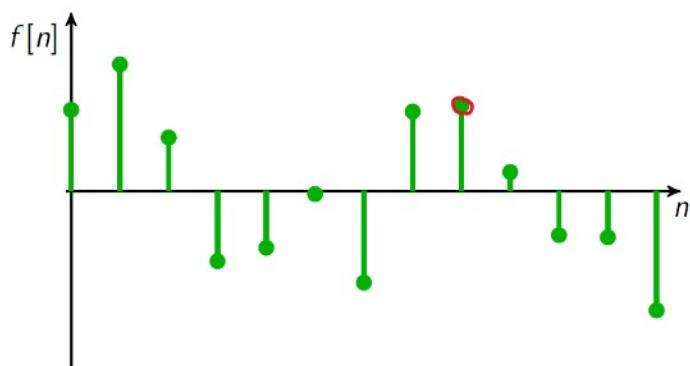
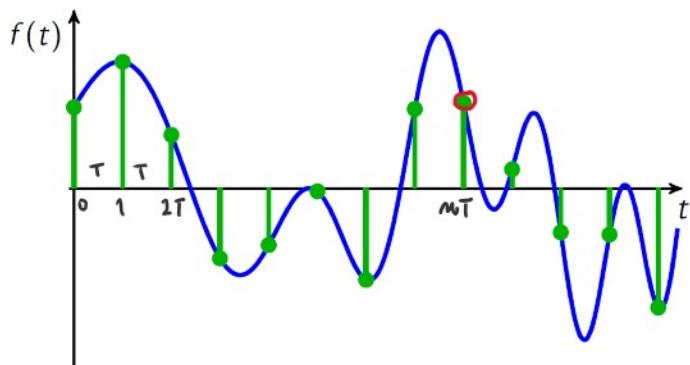
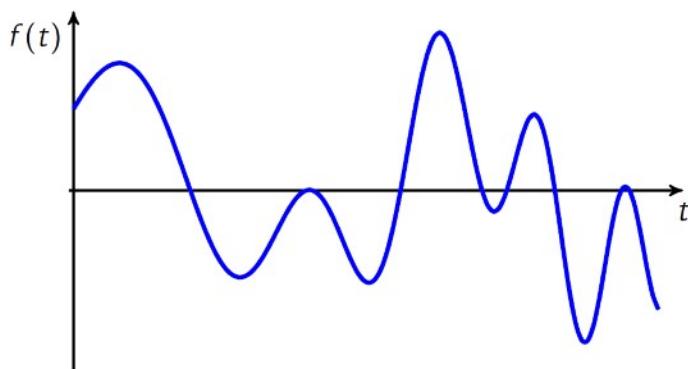


Poznámky k 9. přednášce



Laplacova tr.

$$F(p) = \int_0^{\infty} f(t) \cdot e^{-pt} dt$$

$$\mathcal{L}\{f^*(t)\} = \sum_{n=0}^{\infty} \int_0^{\infty} \underbrace{\delta(t-nT) \cdot f(n \cdot T)}_{f[n]} \cdot e^{-pt} dt =$$

$$= \sum_{n=0}^{\infty} f[n] \cdot e^{-pnT} \quad z = e^{pT}$$

$$= \sum_{n=0}^{\infty} f[n] \cdot z^{-n}$$

Z-transformace

Z-transformace

$$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n},$$

$$F(z) = \mathcal{Z}\{f[n]\}$$

$$f[n] \xrightarrow{Z} F(z)$$

I. Linearita

$$\mathcal{Z}\{a \cdot f[n] + b \cdot g[n]\} = a \cdot \mathcal{Z}\{f[n]\} + b \cdot \mathcal{Z}\{g[n]\}$$

II. Změna měřítka

$$a^{-n}f[n] = \mathcal{Z}^{-1}\{F(az)\}$$

$$F(a^{-1}z) = \mathcal{Z}\{a^n f[n]\}$$

III. VĚTA O POSUNUTÍ

$$\mathcal{Z}\{f[n-m]\} = z^{-m} \mathcal{Z}\{f[n]\} = z^{-m} F(z) \quad |_{\forall n-m < 0: f[n-m]=0}$$

$$\begin{aligned} \mathcal{Z}\{f[n+m]\} &= z^m \left[\mathcal{Z}\{f[n]\} - \sum_{\nu=0}^{m-1} f[\nu] z^{-\nu} \right] \\ &= z^m \left[F(z) - \sum_{\nu=0}^{m-1} f[\nu] z^{-\nu} \right] \end{aligned}$$

pr.

$$\mathcal{Z}\{f[n-2]\} = \left\{ \text{nu} = 2 \right\} = z^{-2} F(z)$$

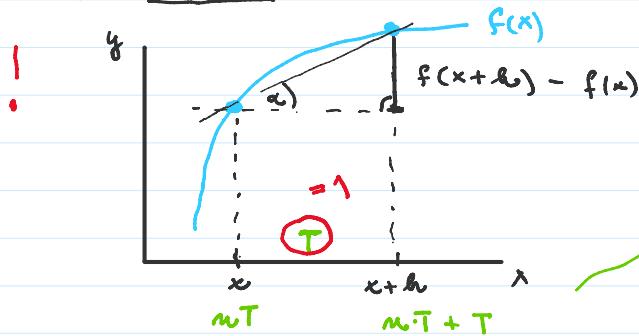
$$\begin{aligned} \mathcal{Z}\{f[n+2]\} &= z^2 \left[F(z) - \sum_{k=0}^{2-1=1} f[k] \cdot z^k \right] = z^2 \left[F(z) - f[0] \cdot z^0 - f[1] \cdot z^1 \right] \\ &= z^2 \left[F(z) - f[0] \cdot 1 - f[1] \cdot z^1 \right] = z^2 F(z) - z^2 f[0] - z \cdot f[1] \end{aligned}$$

IV. Cístečný součet

$$\mathcal{Z} \left\{ \sum_{\nu=0}^n f[\nu] \right\} = \frac{z}{z-1} F(z)$$

$$\mathcal{Z} \left\{ \sum_{\nu=0}^{n-1} f[\nu] \right\} = \frac{1}{z-1} F(z)$$

V. Difference



$$+g \propto = \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$f'(mT) \approx \frac{f(mT+T) - f(mT)}{T}$$

$$\Delta f[m] = \frac{f[m+1] - f[m]}{1}$$

$$\Delta^0 f[n] = f[n]$$

$$\Delta^1 f[n] = f[n+1] - f[n]$$

$$\begin{aligned}\Delta^2 f[n] &= \Delta'(\Delta' f[n]) = \Delta' (f[n+1] - f[n]) = \\&= f[n+2] - f[n+1] - (f[n+1] - f[n]) = \\&= f[n+2] - 2 \cdot f[n+1] + f[n]\end{aligned}$$

$$\Delta^m f[n] = \Delta^1 [\Delta^{m-1} f[n]]$$

$$\mathcal{Z}\{\Delta^m f[n]\} = \mathcal{Z}\{f[n+1] - f[n]\} = \mathcal{Z}\{f[n+m]\} = z^m \left[\mathcal{Z}\{f[n]\} - \sum_{\nu=0}^{m-1} f[\nu]z^{-\nu} \right] = z^m \left[F(z) - \sum_{\nu=0}^{m-1} f[\nu]z^{-\nu} \right]$$

$$\begin{aligned}
 &= z^1 \left[F(z) - \sum_{k=0}^0 f[k] \cdot z^{-k} \right] - F(z) = z \cdot \left[F(z) - f[0] \cdot z^{-0} \right] - F(z) = \\
 &\quad \cancel{\{f[n+1]\}} \quad \cancel{z^0} \quad \cancel{1} \\
 &= z \cdot \tilde{F}(z) - z \cdot f[0] - F(z) = (z-1)F(z) - z \cdot f[0]
 \end{aligned}$$

$$\mathfrak{I} \{ \Delta^2 f[n] \} = \mathfrak{I} \{ f[n+2] - 2 \cdot f[n+1] + f[n] \} =$$

$$= \bar{z}^2 \left[F(z) - \sum_{k=0}^1 f[k] \cdot \bar{z}^k \right] - 2 \bar{z} \{ f[m+1] \} + F(z) =$$

$$= \frac{1}{2} \left[f(z) - f[0] \cdot z^0 - f[1] \cdot z^1 \right] - 2 \cdot \frac{1}{2} \{ f[u+1] \} + f(z) =$$

$$= \underline{z^2 F(z)} - z^2 \cdot f[0] - z \underline{f[1]} - 2 \left(\underline{z^2 F(z)} - z f[0] \right) + \underline{F(z)} = \\ = (z^2 - 2z + 1) F(z) - z^2 f[0] - z f[1] + 2z f[0] = \dots$$

$$= (z^2 - 2z + 1) f(z) - z^2 f[0] - z f[1] + 2z f[0] = \dots$$

$$= (z^2 - 2z + 1) f(z) - z^2 f[0] - z f[1] + 2z f[0] = \dots$$

VII. Konvolut

$$\mathcal{Z}\{f[n] * g[n]\} = \mathcal{Z}\left\{\sum_{m=0}^{\infty} f[n-m] \cdot g[m]\right\} = F(z) \cdot G(z)$$

VII. Derivace obrazu

$$\mathcal{Z}\{nf[n]\} = -z \frac{dF(z)}{dz}$$

Tabulky:

$f(n) = \mathcal{Z}^{-1}\{F(z)\}$	$F(z) = \mathcal{Z}\{f(n)\}$	
$\delta(n)$	1	1
$\mathbf{1}(n)$	$\frac{1}{1-z^{-1}}$	$\frac{z}{z-1}$
a^n	$\frac{1}{1-az^{-1}}$	$\frac{z}{z-a}$
na^{n-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$	$\frac{z}{(z-a)^2}$
$(n+1)a^n$	$\frac{1}{(1-az^{-1})^2}$	$\frac{z^2}{(z-a)^2}$
n	$\frac{z^{-1}}{(1-z^{-1})^2}$	$\frac{z}{(z-1)^2}$
n^2	$\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$	$\frac{z(z+1)}{(z-1)^3}$

$$\begin{aligned} \frac{1}{1-z^{-1}} &= \frac{1}{1-\frac{1}{z}} = \\ \Rightarrow \frac{1}{\frac{z-1}{z}} &= \frac{z}{z-1} \end{aligned}$$

$$y[n+2] + a \cdot y[n+1] + b \cdot y[n] = u[n] \quad \dots \quad Y(z) = \frac{Q(z)}{N(z)} \quad \dots \quad y[n] = \sum_{k=0}^{-1} \{Y(z)\}$$

Nabídka - popis

$$n[k] = C \cdot c[k-1] + A \cdot x[k]$$

$$p[k] = -D \cdot c[k] + B \cdot x[k]$$

$$n[k] = p[k]$$

$$C \cdot c[k-1] + A \cdot x[k] = -D \cdot c[k] + B \cdot x[k]$$

$$C \cdot c[k-1] + D \cdot c[k] = B \cdot x[k] - A \cdot x[k]$$

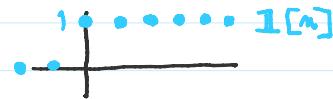
$$\left\{ \begin{array}{l} q[n] = c[k] \\ \frac{B-A}{D} = \alpha \end{array} \right.$$

$$y[n] + \beta \cdot y[n-1] = \alpha \cdot x[k]$$

$$x[k] = 1[n]$$

$$y[-1] = 0$$

$$y[n] + \beta \cdot y[n-1] = \alpha \cdot 1[n]$$



$$y[n] + \beta \cdot y[n-1] = \alpha \cdot 1[n]$$



a) $n=0: y[0] + \beta \cdot y[-1] = \alpha \cdot 1[0]$
 $y[0] = -\beta \cdot y[-1] + \alpha \cdot 1[0] = \alpha$

$n=1: y[1] + \beta \cdot y[0] = \alpha \cdot 1[1]$
 $y[1] = -\beta \cdot y[0] + \alpha \cdot 1[1] = -\beta \cdot \alpha + \alpha = \alpha(1-\beta)$

$n=2: y[2] + \beta \cdot y[1] = \alpha \cdot 1[2]$
 $y[2] = -\beta \cdot y[1] + \alpha \cdot 1[2] = -\beta(\alpha(1-\beta)) + \alpha =$
 $= -\beta(\alpha - \alpha\beta) + \alpha =$
 $= \alpha\beta^2 - \alpha\beta + \alpha = (\beta^2 - \beta + 1)\alpha$

$\sum z^n =$
 $= \frac{1-z^{n+1}}{1+z}$

$$y[n] = \alpha(1-\beta + \beta^2 - \beta^3 + \beta^4 \dots (-\beta)^n) = \alpha \sum_{m=0}^n (-\beta)^m =$$
 $= \alpha \frac{1 - (-\beta)^{n+1}}{1 + \beta} = \frac{\alpha}{1 + \beta} - \frac{\alpha(-\beta)^{n+1}}{1 + \beta} = \frac{\alpha}{1 + \beta} + \frac{\alpha\beta}{1 + \beta} \cdot (-\beta)^n$

$$\underline{y[n] = \frac{\alpha}{1+\beta} + \frac{\alpha\beta}{1+\beta} \cdot (-\beta)^n}$$