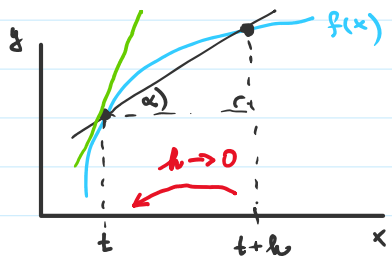
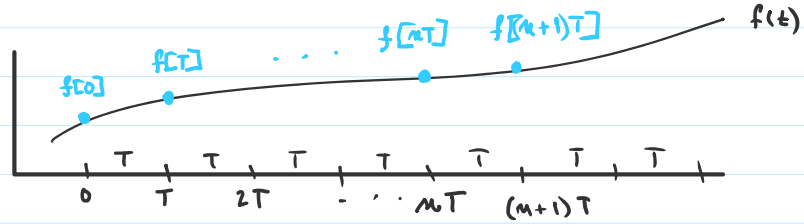
Poznámky ke 12. přednášce

I. Diskretizace

$$y''(t) + a\cdot y'(t) + b\cdot y(t) = u(t) \quad \Rightarrow \quad y[n+2] + a\cdot y[n+1] + b\cdot y[n] = u[n]$$



$$\operatorname{tg}\alpha = \frac{f(t+h) - f(t)}{h} \qquad f'(t) \approx \lim_{h\to 0} \frac{f(t+h)-f(t)}{h}$$



$$f'(t) \sim \frac{f[(n+1)T] - f[nT]}{T}$$

$$f'(t) \sim \underbrace{\frac{f[n+1] - f[n]}{T}}_{\text{Eulerova dopředná diference}}$$

$$f[nT] \sim f[n]$$
$$f[(n+1)T] \sim f[n+1]$$

$$\Delta_1 f = f[n+1] - f[n]$$
$$\Delta_2 f = \Delta_1(\Delta_1 f) = \Delta_1(f[n+1] - f[n]) = f[n+2] - f[n+1] - (f[n+1] - f[n])$$
$$= f[n+2] - 2f[n+1] + f[n]$$

$$f'(t) \approx \frac{f[n+1] - f[n]}{h} + \varepsilon$$

$$f''(t) \approx \frac{f[n+2] - 2f[n+1] + f[n]}{h^2} + \varepsilon$$

PŘÍKLAD:

$$y'(t) = -y(t) \; ; \quad y(0) = 1$$

$$p\cdot Y(p) - y(0) = -Y(p)$$
$$p\cdot Y(p) - 1 = -Y(p)$$
$$p\cdot Y(p) + Y(p) = 1$$
$$Y(p)(p+1) = 1$$

$$Y(p) = \frac{1}{p+1}$$
$$\Downarrow$$
$$y(t) = e^{-t}$$

$$y'(t) \approx \frac{y[n+1] - y[n]}{h}$$

$$y[n+1] - y[n] \qquad h\cdot[\ldots]$$

```
clear;
h = 0.5;        % časová diference
t = [0:h:1];
yn(1)=1;
```

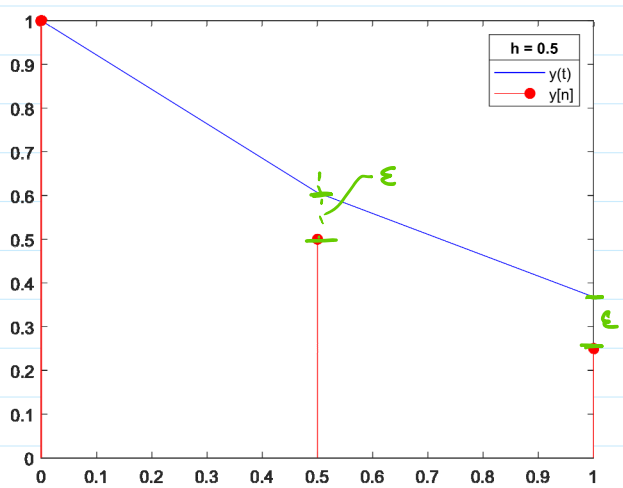
$$\frac{y[n+1]-y[n]}{h} = -y[n]$$

$$y[n+1]-y[n] = -h\cdot y[n]$$

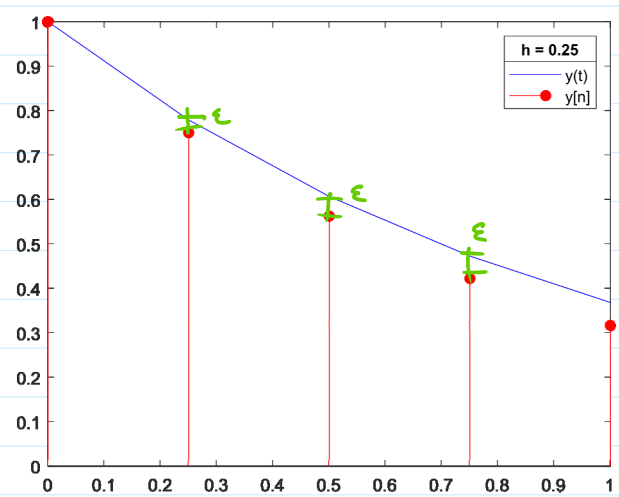
$$y[n+1] = y[n] - h\cdot y[n]$$

```
t = [0:h:1];
yn(1)=1;
yt(1)=1;
% spojitý systém yt
% diskretizovaný systém yn
yt = exp(-t);

for i=1:(length(t)-1)
   yn(i+1) = yn(i) - h*yn(i);
end

figure(1);
plot(t,yt,'blue');
hold on
stem(t,yn,'filled','red');
lgd = legend('y(t)','y[n]');
title(lgd,'h = 0.5');
```

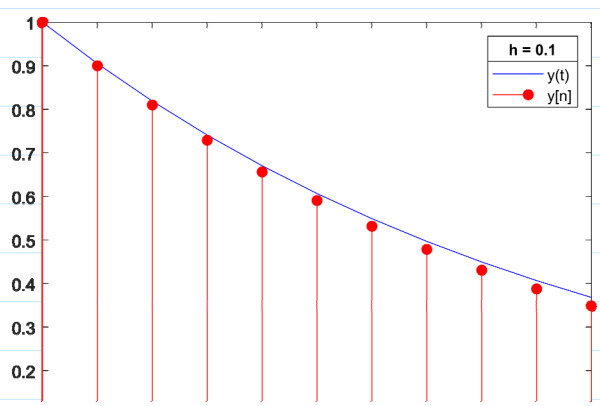
a) $h = 0.5$



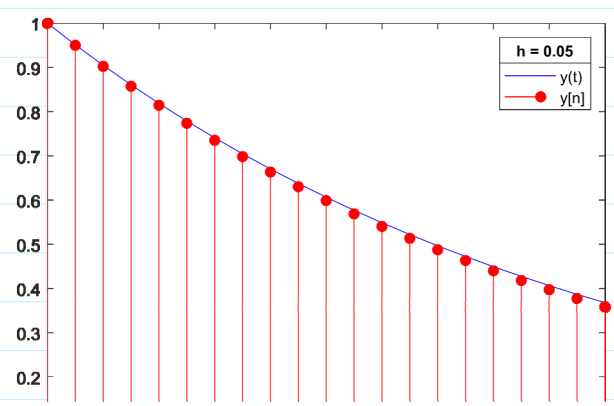
b) $h = 0{,}25$

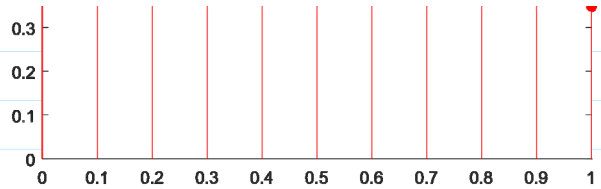


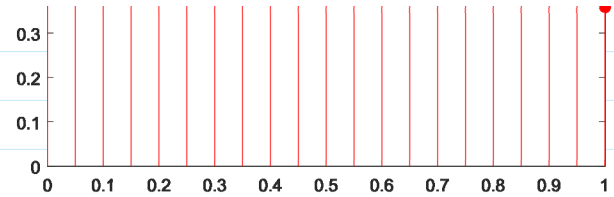
c) $h = 0{,}1$



d) $h = 0{,}05$







Příklad 2 :

$$y''(t) + 2\cdot y'(t) + y(t) = \mathbb{1}(t) \quad ; \quad y(0) = 1 \quad ; \quad y'(0) = -1$$ ← Diferenciální r.

$$\begin{cases} y(t) \to y[n] \\ \mathbb{1}(t) \to \mathbb{1}[n] \end{cases}$$

$$y'(t) = \frac{y[n+1] - y[n]}{h}$$

$$y''(t) = \frac{y[n+2] - 2y[n+1] + y[n]}{h^2}$$

$$\frac{y[n+2] - 2y[n+1] + y[n]}{h^2} + 2\cdot\frac{y[n+1] - y[n]}{h} + y[n] = \mathbb{1}[n]$$

$$y[n+2] - 2y[n+1] + y[n] + 2h\,y[n+1] - 2h\,y[n] + h^2 y[n] = h^2\,\mathbb{1}[n]$$

$$y[n+2] + (2h-2)\,y[n+1] + (h^2 - 2h + 1)\,y[n] = h^2\cdot\mathbb{1}[n]$$

Diferenční rovnice

Eulerova zpětná diference

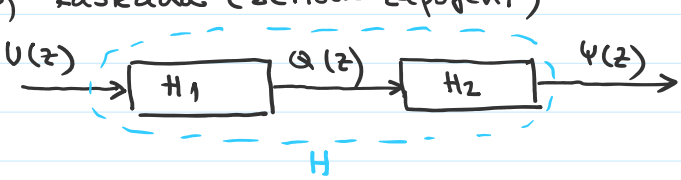
$$f'(t) \approx \frac{f[n] - f[n-1]}{h}$$

$$f''(t) \approx \Delta_1\left(\frac{f[n] - f[n-1]}{h}\right) = \frac{f[n] - f[n-1] - (f[n-1] - f[n-2])}{h^2} =$$

$$= \frac{f[n] - 2f[n-1] + f[n-2]}{h^2}$$

II. Subsystémy

a) kaskáda (seriové zapojení)



$$H(z) = \frac{Y(z)}{U(z)}$$

$$H_1(z) = \frac{Q(z)}{U(z)}$$

$Y(t)$ $\quad H_2(z)\cdot Q(z)$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{H_2(z)\cdot Q(z)}{\frac{Q(z)}{H_1(z)}} =$$

$$= \frac{H_2(z)\cdot \cancel{Q(z)}}{1} \cdot \frac{H_1(z)}{\cancel{Q(z)}}$$

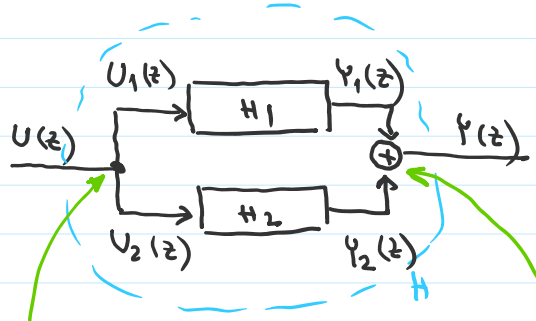
$$H(z) = H_1(z)\cdot H_2(z)$$

$$H_1(z) = \frac{Q(z)}{U(z)}$$

$$H_2(z) = \frac{Y(z)}{Q(z)}$$

$$\begin{cases} U(z) = \dfrac{Q(z)}{H_1(z)} \\ Y(z) = H_2(z)\cdot Q(z) \end{cases}$$

b) Paralelní zapojení



$$H(z) = \frac{Y(z)}{U(z)}$$

$$H_1(z) = \frac{Y_1(z)}{U_1(z)}$$

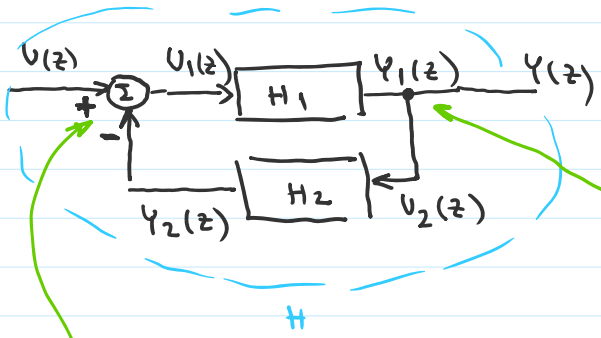
$$H_2(z) = \frac{Y_2(z)}{U_2(z)}$$

$$U(z) = U_1(z) = U_2(z) \qquad Y(z) = Y_1(z) + Y_2(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{Y_1(z) + Y_2(z)}{U(z)} = \frac{Y_1(z)}{U(z)} + \frac{Y_2(z)}{U(z)} = \frac{Y_1(z)}{U_1(z)} + \frac{Y_2(z)}{U_2(z)}$$

$$H(z) = H_1(z) + H_2(z)$$

c) zpětná vazba (záporná)



$$H(z) = \frac{Y(z)}{U(z)}$$

Výstup: $Y_1(z) = Y(z) = U_2(z)$

Vstup: $U_1(z) = U(z) - Y_2(z)$

$$H_1(z) = \frac{Y_1(z)}{U_1(z)} = \frac{Y(z)}{U_1(z)} \Rightarrow Y(z) = H_1(z)\cdot U_1(z)$$

$$Y(z) = H_1(z)\left(U(z) - Y_2(z)\right) =$$

$$H_1(z) = \frac{Y_1(z)}{U_1(z)} = \frac{\quad}{U_1(z)} \Rightarrow Y(z) = H_1(z)\cdot U_1(z)$$

$$H_2(z) = \frac{Y_2(z)}{U_2(z)}$$

$$\Rightarrow Y_2(z) = H_2(z)\cdot U_2(z)$$

$$Y(z) = H_1(z)\left(U(z) - Y_2(z)\right) =$$

$$= H_1(z)\left[U(z) - H_2(z)\cdot U_2(z)\right] =$$

$$= H_1(z)\left[U(z) - H_2(z)\cdot Y(z)\right]$$

$$Y(z) = H_1(z)\left[U(z) - H_2(z)\cdot Y(z)\right]$$

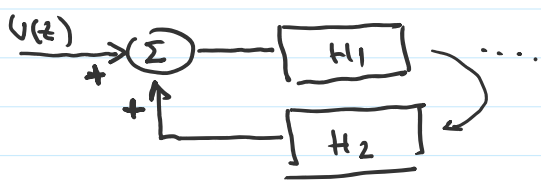
$$Y(z) = H_1(z)\cdot U(z) - H_1(z)\cdot H_2(z)\cdot Y(z)$$

$$Y(z) + H_1(z)H_2(z)\cdot Y(z) = H_1(z)\cdot U(z)$$

$$Y(z)\left[1 + H_1(z)H_2(z)\right] = H_1(z)\cdot U(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{H_1(z)}{1 + H_1(z)H_2(z)}$$

Kladná zpětná vazba



$$H(z) = \frac{H_1(z)}{1 - H_1(z)H_2(z)}$$