Experiment Nr. 23

Determination of resistances using a Wheatstone bridge

Theoretical part

In the world of analogue electronics, we come across various signals, some of them are measured by changes in resistance and some of them are with changes in inductance and capacitance. If we consider the resistance, most of the industrial sensors like temperature, strain, humidity, displacement, liquid level, etc. produces the change in value of the resistance for an equivalent change in the respective quantity. Therefore, there is a need for a signal conditioning for every resistance-based sensor.

Generally, the resistance measurement is divided into three basic ranges. If the resistance measurement is possibly from a few μΩ to mΩ, then it is considered as a low resistance measurement. This measurement is actually used for research purpose. If the measurement is from 1 Ω to few hundreds of kΩ is generally referred as a medium resistance measurement. Measurement of normal resistors, potentiometers, thermistors, etc. comes under this category. Very high resistance measurement is considered from few MΩ to greater than 100 MΩ. For finding the medium value of the resistance different direct or indirect methods are used, but mostly Wheatstone bridge is used.

A Wheatstone Bridge is essentially an electrical circuit designed to compare resistances or measure the unknown value of a resistor's resistance by generating a balance between the bridge circuit's two legs. A typical diagram of the Wheatstone bridge is shown in Fig. 1.

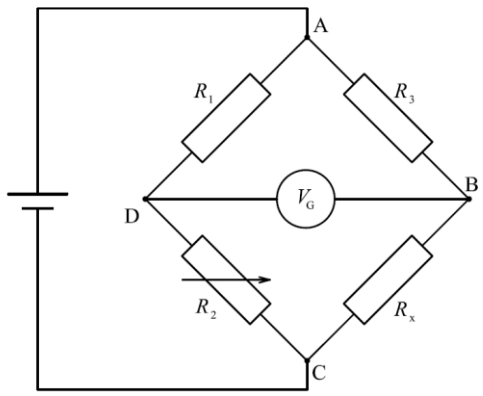

Fig. 1 Schematic diagram of the Wheatstone bridge

It is often referred to as a "resistance bridge". To calculate the value of unknown resistance, Wheatstone bridge employs the comparison approach.

The Wheatstone bridge is made up of four resistors connected in a diamond-shaped circuit. Although it was designed to measure the value of unknown resistance, it may now be used to calibrate measuring instruments such as voltmeters and ammeters using changing resistance and a simple mathematical calculation. The Wheatstone bridge circuit provides an extremely accurate value for the measured resistance.

Wheatstone bridges are simple series-parallel resistor arrangements that are connected between a supply voltage and ground. Wheatstone bridge circuit has two input terminals A, C and two output terminals B, D with four resistors R_1, R_2, R_3, R_x (see Fig. 1). The bridge is used to precisely determine the unknown resistance R_x by comparing it to a known value of resistance.

The null or balanced condition is utilized to find the resistance in this circuit. To achieve a balanced condition, the voltages at B and D terminals must be equal. In order for no current to flow through the meter and a balanced condition to be achieved, one of the resistors must be variable.

Its operation is as follows: A balanced voltage between the sites allows the measurement by using the ratio of three known resistors R_1, R_2, R_3 to measure the fourth unknown resistance R_x. To balance the device, the resistors are changed to adjustable variable resistors, and the mathematical ratio is employed to determine the fourth resistance:

$$\frac{R_2}{R_1} = \frac{R_x}{R_3} \Rightarrow R_x = \frac{R_2 R_3}{R_1}$$

Theory of the measurement principle and measurement procedure

The experiment puts the power voltage on a one-meter long pilot wire with constant width. The wire ends are connected to the unknown resistance R and another, series connected resistance R_S that is variable but known precisely (see Fig. 2).

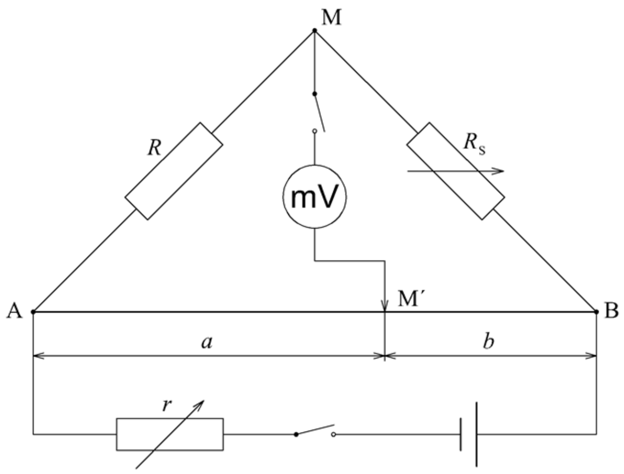

Fig. 2 Wheatstone bridge circuit used in our experiment

A sliding contact M´ splits the pilot wire into two sections of lengths a and b (with corresponding wire resistances). The sliding contact is connected to the cross-point M between R_S and R through a sensitive millivoltmeter inserted as a null indicator. The value of the resistance r could change the bridge sensitivity only. If the voltage on the millivoltmeter is aligned on zero, then:

$$\frac{R}{R_S} = \frac{a}{b}$$

Thus, the unknown resistance R could be determined as

$$R = \frac{a}{b} R_S = \frac{a}{l - a} R_S$$

where $l = a + b$.
In our experiment, we will determine the resistance of selected resistors. Additionally, the resistor material property, resistivity ρ, will be calculated. Resistivity ρ could be determined from the following formula:

$$R = \rho \frac{l}{S}, \ [\rho] = \Omega \cdot \mathrm{m}$$

where l is the resistor wire length, S is the resistor wire cross-section area.

Measurement objectives

1. Determine the resistance of two resistors using ohmmeter and calculate the uncertainty.

2. Arrange the Wheatstone bridge wiring and determine the resistance of two resistors with uncertainty.

3. Calculate the material resistivity of the resistors and determine the uncertainty.

Comments

Ad 2. The best measurement accuracy for the experimental configuration is achieved with a symmetric assembly, i.e. if the sliding contact is positioned in the middle of the pilot wire, so that both sections a and b have the same length. However, the resistance decade used in our experiment as the R_S resistor allows adjusting of discrete values only. Thus, we will find the most accurate R_S value to balance the bridge roughly and the fine balancing will be carried out by sliding the contact in the middle of the pilot wire, so that the $\frac{a}{b}$ ratio won't equal 1.
The sensitivity resistor r should be adjusted to 3 kΩ. The power voltage should be about 4.5 V.

Ad 3. The material resistivity could be calculated from the resistance R and following wire parameters:

Wire Material	Wire length [m]	Wire diameter [mm]
Cu	9.9	0.20
Fe	32.0	0.36
Constantan	55.5	0.40

Uncertainty calculation notes

Resistance *R*

The relative uncertainty of the resistance determined from the Wheatstone bridge balancing could be calculated as follows:

$$u_{rR} = \sqrt{4u_{raB}^2 + u_{rR_SB}^2}$$

where u_{rR_SB} is given by the producer of the resistance decade (stated on the decade box) and the u_{raB} is given by the linear scale reading error.
The resistance determined by the direct measurement using ohmmeter has the error level given by the digital instrument producer. The user's manual shows the standard errors according to the selected quantities and ranges as follows:

<table>
<tr><th>Quantity</th><th>Range</th><th>standard error</th></tr>
<tr><td rowspan="4">Direct Current</td><td>2 mA</td><td rowspan="3">0.3 % rdg + 3 digits</td></tr>
<tr><td>20 mA</td></tr>
<tr><td>200 mA</td></tr>
<tr><td>20 A</td><td>0.5 % rdg + 3 digits</td></tr>
<tr><td rowspan="4">Direct Voltage</td><td>200 mV</td><td rowspan="4">0.05 % rdg + 3 digits</td></tr>
<tr><td>2 V</td></tr>
<tr><td>20 V</td></tr>
<tr><td>200 V</td></tr>
<tr><td rowspan="2">Resistance</td><td>200 Ω</td><td>0.2 % rdg + 5 digits</td></tr>
<tr><td>2 kΩ and higher</td><td>0.15 % rdg + 3 digits</td></tr>
</table>

Resistivity *ρ*
The standard uncertainty of the resistivity is given by the law of summation in quadrature (see the theory of uncertainty calculation). The wire length was determined with an accuracy of 5 cm and the wire diameter was measured using micrometre.