Related Topics

Maxwell's equations, electrical eddy field, magnetic field of coils, magnetic flux, induced voltage

Principle

A magnetic field of variable frequency and varying strength is produced in a long coil. The voltages induced across thin coils which are pushed into the long coil are measured depending on the frequency of the current in the field coil and the strength of its magnetic field as well as the number of turns and the diameter of the induction coil.

Equipment

1	Field coil, 750 mm, 485 turns/m	11001-00
1	Induction coil, 300 turns, d = 40 mm	11006-01
1	Induction coil, 300 turns, d = 32 mm	11006-02
1	Induction coil, 300 turns, d = 25 mm	11006-03
1	Induction coil, 200 turns, d = 40 mm	11006-04
1	Induction coil, 100 turns, d = 40 mm	11006-05
1	Induction coil, 150 turns, d = 25 mm	11006-06
1	Induction coil, 75 turns, d = 25 mm	11006-07
1	Digital function generator	13654-99
2	Multi-range meter, analogue	07026-00
3	Connecting cord, l = 750 mm, red	07362-01
2	Connecting cord, l = 750 mm, blue	07362-04

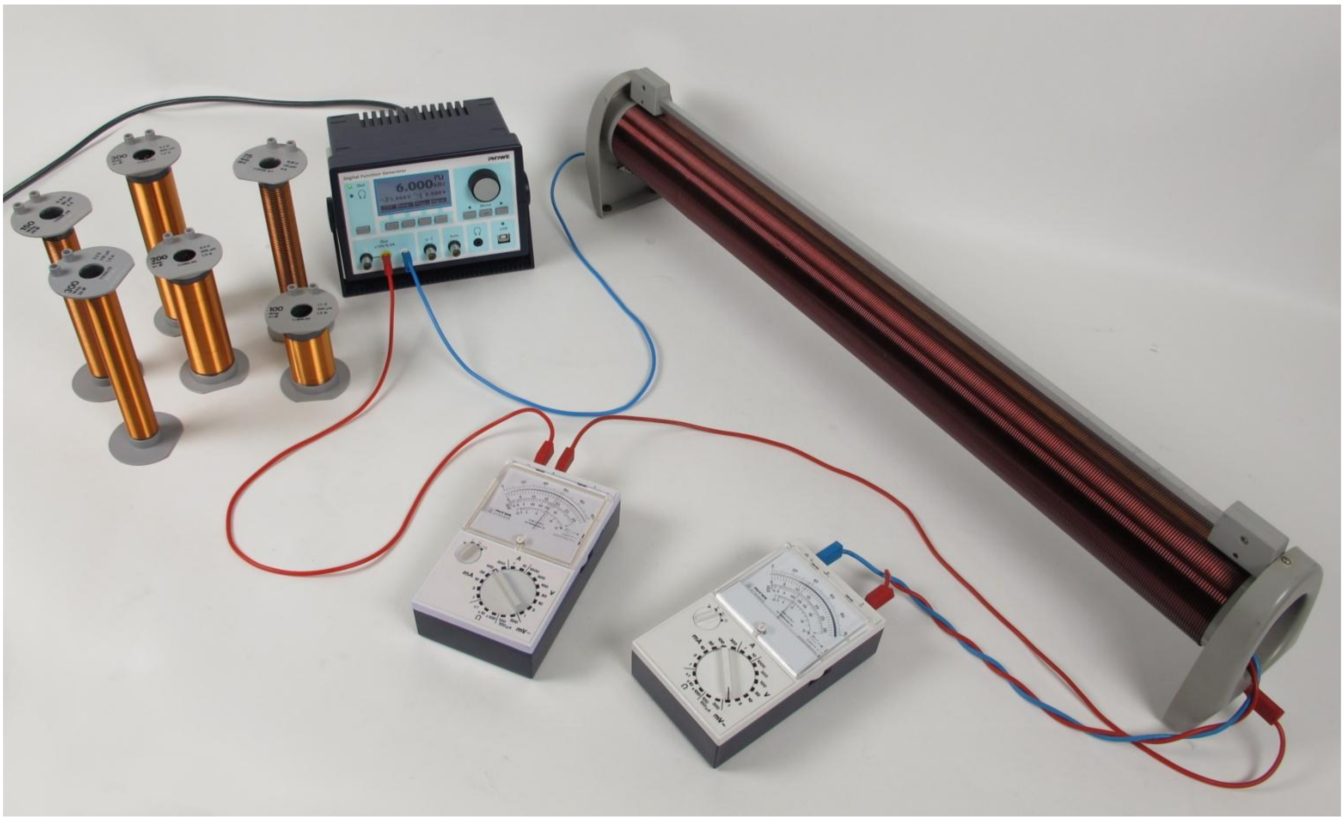

Fig. 1: Experimental set-up with one induction coil slid into the field coil.

Tasks

Measure the induction voltage as a function of

1. the current in the field coil at a constant frequency,
2. the frequency of the magnetic field at a constant current,
3. the number of turns of the induction coil at constant frequency and current and
4. the cross-sectional area of the induction coil at constant frequency and current.

Set-up

The experimental set-up is as shown in Fig. 1. One multi-range meter is set up in series connection to the field coil and the digital function generator in order to measure the current in the field coil. The second multi-range meter is connected to the induction coil to measure the induced voltage.

Procedure

Set both multi-range meter to AC (alternating current) mode. For the measurements choose ranges up to 300 mA and up to 3 V respectively. Be careful to read the correct scales for the measurements.

For a detailed description of the operation of the digital function generator please refer to the manual.

Task 1:

Tune the current in the field coil by turning up the amplitude of the sinus signal of the digital function generator. Start with an amplitude of 0.5 V and increase to a maximum of 10 V in steps of 0.5 V.

Task 2:

Choose the current in the field coil between 20 mA and 40 mA. The effect of frequency should be studied between 1 kHz and 12 kHz, since below 0.5 kHz the coil practically represents a short circuit and above 12 kHz the accuracy cannot be guaranteed. Increase the frequency in steps of 0.5 kHz. In order to maintain a constant current in the field coil for various frequencies, you have to adjust the amplitude of the sinus signal very accurately for each frequency.

Tasks 3 and 4:

Choose frequency and signal-amplitude and maintain these settings throughout the measurements. Note down the induced voltage, number of turns and diameter for each induction coil.

Theory

To understand the fundamentals of this experiment, two cases must be considered. First we treat the temporal variation of the magnetic flux through an area which induces a voltage in a conductor. In this experiment, this voltage will be measured. Second the temporal variation of a current in a conductor, which induces a magnetic field, will be regarded whereby the current is the second measurand.

The temporal variation of the magnetic flux leads to Faraday's law of induction. The magnetic flux ϕ through an area A is obtained by integrating the magnetic flux density $\vec{B}$ over this area (1):

$$\phi = \int_A \vec{B}\ \mathrm{d}\vec{A} \qquad (1)$$

After the law of induction the temporal variation of the flux ϕ induces the voltage U_{ind} (2). Considering the flux ϕ through A which is enclosed by a conductor loop, the induced voltage is the integral of the electric field $\vec{E}$ in the conductor loop over the area's boundary C:

$$U_{\mathrm{ind}} = -\frac{\partial \phi}{\partial t} = \oint_C \vec{E}\ \mathrm{d}\vec{s} \qquad (2)$$

This relationship for one conductor loop is the second of Maxwell's equations.

For n parallel conductor loops equation (3) is valid, if ϕ is the same for all loops:

$$U_{\text{ind}} = -n \cdot \frac{\partial \phi}{\partial t} \quad (3)$$

Now we have a relation between the induced voltage and the number of coils. We need to find a way to calculate the right side of equation (3). Therefore we consider the magnetic flux density in a long coil, which is constant, so that equation (1) simplifies to the following relation:

$$\phi = \vec{B} \int_A \mathrm{d}\vec{A} = \vec{B} \cdot \vec{A} \quad (4)$$

Utilizing Maxwell's first equation, we can find an expression for $\vec{B}$, which depends only on measurands and fundamental constants.

Maxwell's first equation (5) states that in a conductor a current generates a magnetic field, of which the closed field lines circle around the currents:

$$\int_{A'} \vec{J} \cdot \mathrm{d}\vec{A} = {}^{1}/_{\mu} \cdot \oint_{C'} \vec{B} \, \mathrm{d}\vec{s} \quad (5)$$

There μ is the magnetic conductivity (a material's constant), C' is the inductor coil which is penetrated by the field coil's magnetic flux density $\vec{B}$ and encloses the area A'. In the field coil flows a current I which is given by integrating the inductor coil's area A over the current density $\vec{J}$, so $I = \int_A \vec{J} \cdot \mathrm{d}\vec{A}$.

For a long coil with n turns, the absolute value of $\vec{B}$ can be approximated by the following equation:

$$B = n \cdot \mu \cdot {}^{I}/_{l} \quad (6)$$

There, l is the length of the coil which must be significant higher than the diameter. In air, μ can be approximated by the magnetic constant

$$\mu_0 = 4\,\pi \cdot 10^{-7}\,\mathrm{V \cdot s \cdot A^{-1} \cdot m^{-1}}. \quad (7)$$

If an alternating current $I(t) = I_0 \cdot \sin(\omega t)$ with the frequency $\nu = {}^{\omega}/_{2\pi}$ flows through the field coil, then from (6) the flux density in the field coil is a function of time and alternates in phase with the current:

$$B(t) = \frac{n\,\mu_0}{l} \cdot I_0 \cdot \sin(2\pi\,\nu \cdot t) \quad (8)$$

The induced voltage can be calculated by applying equation (6) to equations (3) and (4). Execution of the time derivation gives the following relation for the induced voltage

$$U_{\text{ind}}(t) = -n'A' \cdot \frac{\partial B(t)}{\partial t} = -n'A' \cdot 2\pi\,\nu\,\frac{n\,\mu_0}{l} \cdot I_0 \cdot \cos(2\pi\,\nu \cdot t), \quad (9)$$

where n' is the induction coil's number of turns and A' its cross-sectional area. The induced voltage alternates with the same frequency as the current but is phase-shifted by ${}^{\pi}/_{2}$.

Evaluation and results

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here.

Task 1: Measure the induction voltage as a function of the current in the field coil at a frequency of 10.7 kHz and calculate the magnetic constant μ_0.

In order to vary the magnetic field the current in the field coil has to be altered. With relation (6) the magnetic constant can be calculated if the current and the magnetic field are known. In this experiment the magnetic field is not measured so we need to find another way. Therefore we consider relation (9). As is easily shown (see equation (10)) the magnetic field constant is included in the slope s of the induced voltage's linear dependence of the current.

$$U_{\text{ind}}(t) = -n'A' \cdot 2\pi\,\nu \cdot \frac{n\,\mu_0}{l} \cdot \cos(2\pi\,\nu \cdot t) \cdot I_0 = s(t) \cdot I_0 \quad (10)$$

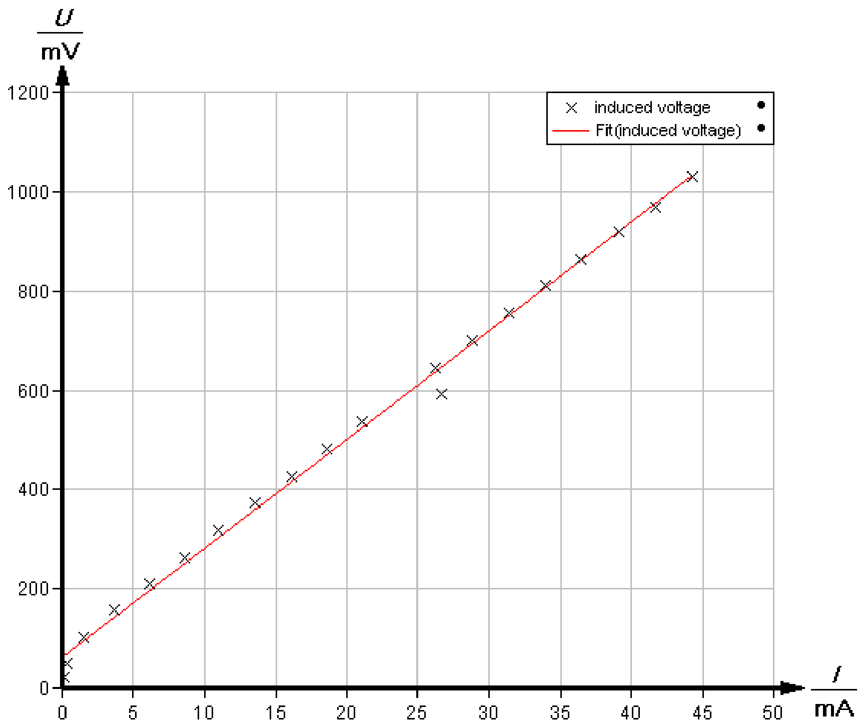

Fig. 2: The graph shows the induced voltage for the coil 11006-01 for varying currents in the field coil. The dependence is linear and follows relation (12).

The time-dependence can be disregarded, if we always measure current and voltage in intervals of one period $= {}^{1}/_{\nu}$. Insertion in relation (10) gives

$$U_{\text{ind}} = -n'A' \cdot 2\pi\nu \cdot \frac{n\,\mu_0}{l} \cdot I_0 \qquad (11)$$

as the cosine-function simply reduces to unity. With equation (11) all other contributions to s are known from the specifics of the used coils and we can calculate μ_0.

Fitting the measurements to a linear function (see Fig. 2) gives equation (12) with the correlation coefficient $R = 0.9982$.

$$U_{\text{ind}}/\text{mV} = 62.2 + 21.9 \cdot I_0/\text{mA} \qquad (12)$$

As can be seen in Fig. 2 for very small fields, the measured values differ from the expected values and the induced voltage tends towards zero. For any currents greater than 3 mA the dependence is indeed linear and the experimental results are well described by equation (12).

For the slope s we obtain from equations (11) and (12):

$$s = n'A' \cdot 2\pi\nu \cdot \frac{n}{l} \cdot \mu_0 = 21.9.$$

From the experiment we calculate $\mu_0 = 1.78 \cdot 10^{-6}\,\frac{\text{Vs}}{\text{Am}}$, which is of the same order as the value given in the literature with $\mu_0^{\text{lit}} = 1.26 \cdot 10^{-6}\,\frac{\text{Vs}}{\text{Am}}$.

Task 2: Measure the induction voltage as a function of the frequency of the magnetic field at constant current.

As can be seen from equation (11) the induced voltage depends linearly on the frequency of the magnetic field and the current in the field coil respectively.

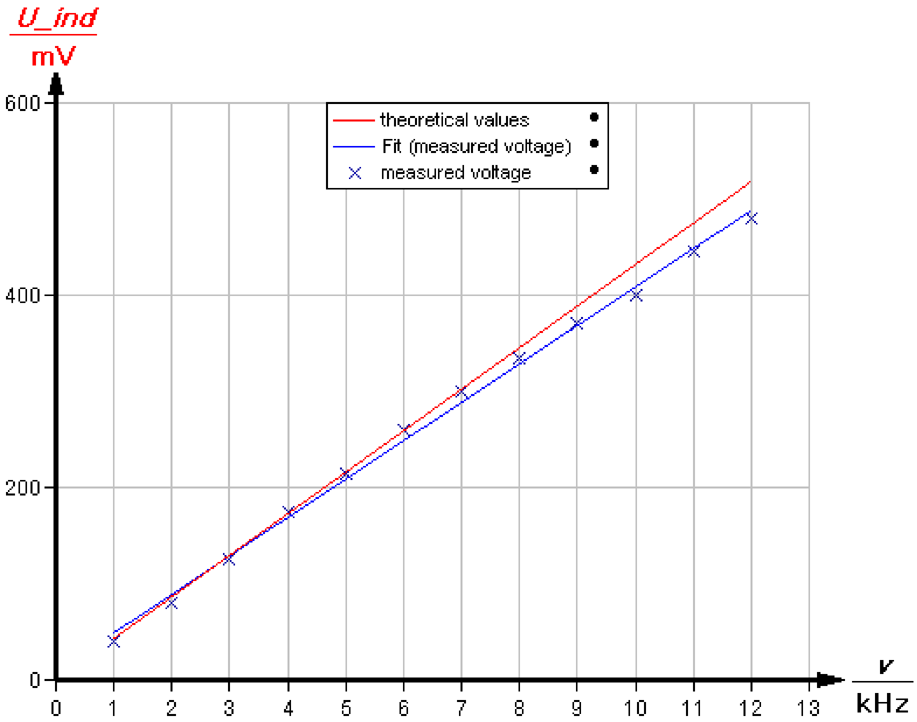

Fig. 3: The graph shows the induced voltage for different frequency at a field strength of 18 μT in the field coil (blue). The dependence follows relation (13). The red line indicates the theoretically expected values.

Measurements were performed at a current of 30 mA which corresponds to a field strength of approximately 18.3 $\mu\mathrm{T}$ in the field coil.

Fitting the measurements to a linear function (see Fig. 3) gives equation (13) with the correlation coefficient $R = 0.9984$ and a slope $s_{\mathrm{fit}} = 40 \pm 8$.

$$U_{\mathrm{ind}}/\mathrm{mV} = 40\ \nu/\mathrm{kHz} + 9 \qquad (13)$$

If we calculate the slope with the known values for the field coil, induction coil and current, we obtain the theoretical value for the slope with

$$s_{\mathrm{calc}} = 43.2\ ,$$

which is well within error limits.

Task 3: Measure the induction voltage as a function of the number of turns of the induction coil at constant frequency and current.

Measurements were done at 10.7 kHz and 30 mA. All induction coils with diameters of 26 mm and 41 mm were studied.

Fitting the measured values for the coils with a diameter $d = 26\ \mathrm{mm}$ to a linear function gives relation (14) corresponding to Fig. 4.

$$U_{\mathrm{ind}}/\mathrm{mV} = 0.899 \cdot n' - 46 \qquad (14)$$

with the correlation $R = 0.9985$.

With a greater diameter we can reach significantly higher induction voltages as Fig. 5 easily shows.

Fitting the measured values for the coils with a diameter $d = 41\,\mathrm{mm}$ to a linear function gives relation (15)

$$U_{\mathrm{ind}}/\mathrm{mV} = 2.78 \cdot n' - 100 \qquad (15)$$

with a slightly better correlation of $R = 0.9989$.

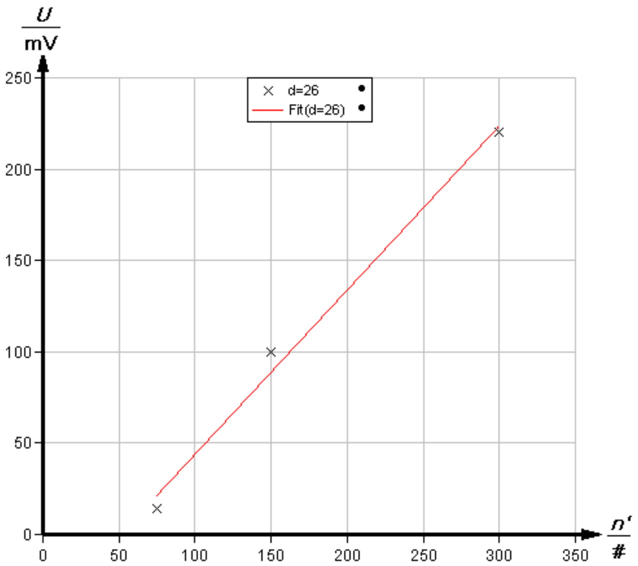

Fig. 4: Induced voltage for different coils with a diameter of 26 mm following relation (14).

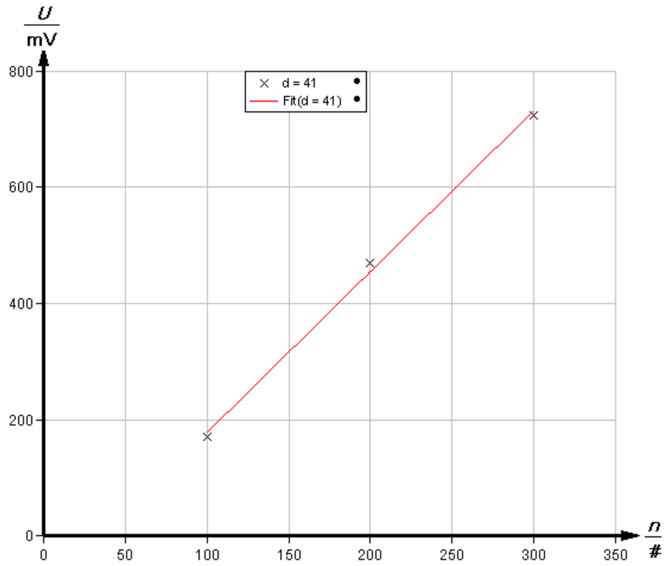

Fig. 5: Induced voltage for different coils with a diameter of 41 mm following relation (15).

Task 4: Measure the induction voltage as a function of the cross-sectional area of the induction coil at constant frequency and current.

The cross-sectional area is the circular area enclosed by the coil. With relation (16) we can calculate the cross-sectional area with the known diameter.

$$A' = {}^{\pi}/_{4} \cdot d^2 \qquad (16)$$

Measurements were done at 10.7 kHz and 30 mA. All induction coils with 300 turns were used in this experiment. Fig. 6 shows the obtained values which were fitted to a linear function as in equation (17).

$$U_{\mathrm{ind}}/\mathrm{mV} = 0.643\,\mathrm{A}/\mathrm{mm}^2 - 100 \qquad (17)$$

The correlation is quite high with $R = 0.9992$.

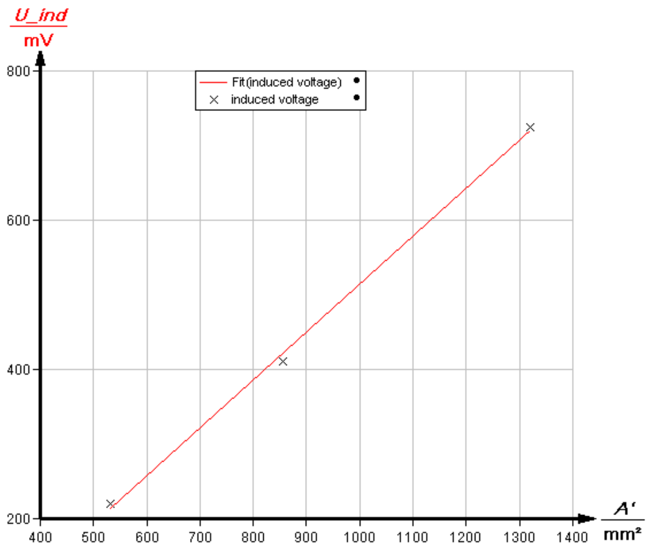

Fig. 6: Induced voltage for various coils with 300 turns. The dependence of the cross-sectional area follows relation (17).