

Theory of uncertainty

measurement - estimation of level of confidence

exact (true) value – hard to get in a real measurement

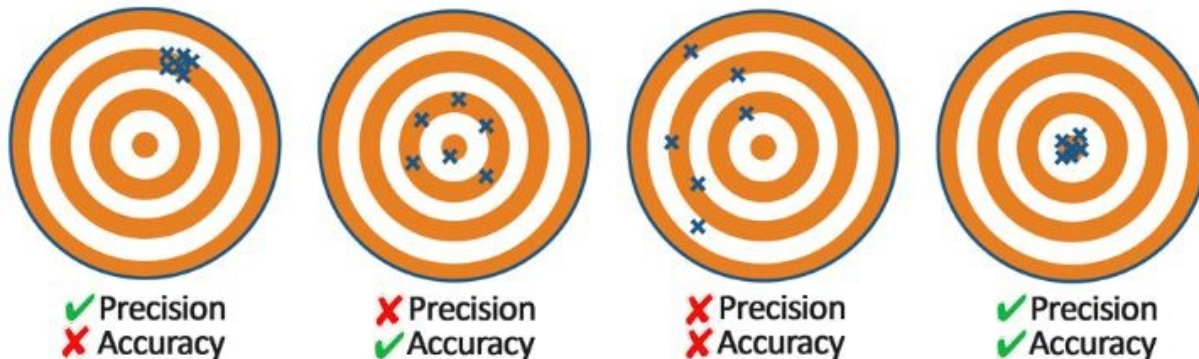
definition of range that surely includes true value

measurement = (best estimate $\pm$ uncertainty) {physical unit}

accuracy = ability to get the result near the true value

precision = ability to get a narrow range of obtained data

true value can be outside this range (inaccurate) !!



Error vs. Uncertainty

measurement errors = difference between the measured value and the 'true value'

measurement uncertainties = quantification of the doubt about the measurement result

Expression of Errors

Absolute Error K_x of a quantity X

same unit as the physical quantity

Relative Error of a quantity X

ratio of absolute error to the value of quantity

$$K_{rx} = \frac{K_x}{x}$$

“r” for “relative”

relative value (no unit)

Types of Errors

Random Errors = statistic fluctuations of results

due to precision limits of the tool (instrument)

could be reduced by averaging of large number of data

Systematic Errors = reproducible inaccuracies

hard to detect, impossible to analyse by statistics

sometimes predictable by instrument calibration

Estimating of the Errors - single measurements

Dimensions: standard error $m_x (= \kappa_x)$

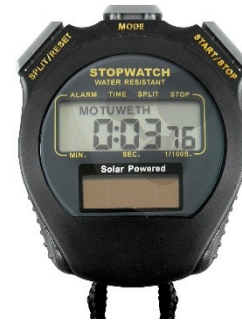
Tape measure: $m_x = 1 \text{ mm}$

Calliper: $m_x = 0.1 \text{ mm}$

Micrometre: $m_x = 0.01 \text{ mm}$



Time: stop-watch standard error $m_x = 0.01 \text{ s}$
human reaction time $m_x = 0.5 \text{ s}$



Electrical quantities – analogue instruments

accuracy class T_p
 $\approx$ maximum relative
error within selected
range $x_{\max}$

$$m_x = \frac{1}{100} T_p x_{\max}$$



Electrical quantities – digital instruments



example from user's manual: $m_x = 0,01\% \text{ rdg.} + 2 \text{ digits}$

base error = percentage of the obtained value (reading)

digits of instrument resolution = dependent on the range

last digital place (considering the decimal place)

e.g. $U = 12.778 \text{ V} \rightarrow \text{one digit} \approx 0.001 \text{ V}$

Estimating of the Errors - repeated measurements

mean (average) value $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

standard deviation of the mean value (data spread)

$$s_{\bar{x}} = \sqrt{\frac{\sum_{i=1}^n (\bar{x} - x_i)^2}{n(n-1)}}$$

Uncertainties

Type A = uncertainty estimated by statistics

Type B = other sources (instrument errors, etc.)

Expression of Uncertainty

Absolute uncertainty u_x of a quantity X

same unit as the physical quantity

Relative Uncertainty u_{rx} of a quantity X

ratio of absolute error to the value of quantity

$$u_{rx} = \frac{u_x}{X}$$

“r” for “relative”

relative value (no unit)

Uncertainty for a Type A evaluation

$$u_{xA} = s_{\bar{x}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Uncertainty for a Type B evaluation

$$u_{zB} = \frac{\Delta z_{\max}}{\chi} \quad \Delta z_{\max} \text{ is the maximum of all detectable systematic errors (= } m_x)$$

$$\chi = \sqrt{3} \quad \text{for the uniform distribution}$$

Combining standard uncertainties A and B Type

law of summation in quadrature

$$u_x = \sqrt{u_{xA}^2 + u_{xB}^2}$$

Combining standard uncertainties – general approach

for quantity $Y = f(X_1, X_1, \dots, X_n)$

$$u_Y = \sqrt{\left(\frac{\partial Y}{\partial X_1}\right)^2 u_{X_1}^2 + \left(\frac{\partial Y}{\partial X_2}\right)^2 u_{X_2}^2 + \dots + \left(\frac{\partial Y}{\partial X_n}\right)^2 u_{X_n}^2}$$

Calculation of sum for Type A and Type B estimation

a set of sub-uncertainties $u_{X_1A}, u_{X_2A} \dots u_{X_nA}, u_{X_1B}, u_{X_2B} \dots u_{X_nB}$

$$u_{YA} = \sqrt{\left(\frac{\partial Y}{\partial X_1}\right)^2 u_{X_1A}^2 + \left(\frac{\partial Y}{\partial X_2}\right)^2 u_{X_2A}^2 + \dots + \left(\frac{\partial Y}{\partial X_n}\right)^2 u_{X_nA}^2}$$

$$u_{YB} = \sqrt{\left(\frac{\partial Y}{\partial X_1}\right)^2 u_{X_1B}^2 + \left(\frac{\partial Y}{\partial X_2}\right)^2 u_{X_2B}^2 + \dots + \left(\frac{\partial Y}{\partial X_n}\right)^2 u_{X_nB}^2}$$

$$u_Y = \sqrt{u_{YA}^2 + u_{YB}^2}$$

Simplification for two quantities in basic operations

values ***a*** and ***b*** are real constants

m and ***n*** are natural indices

Operation	Formula	Uncertainty calculation
Addition or subtraction	$Y = X_1 \pm X_2$	$u_Y = \sqrt{u_{X_1}^2 + u_{X_2}^2}$
	$Y = aX_1 \pm bX_2$	$u_Y = \sqrt{a^2 u_{X_1}^2 + b^2 u_{X_2}^2}$
Multiplication or division	$Y = X_1 X_2$	$u_{rY} = \sqrt{u_{rX_1}^2 + u_{rX_2}^2}$
	$Y = aX_1^m X_2^n$	$u_{rY} = \sqrt{m^2 u_{rX_1}^2 + n^2 u_{rX_2}^2}$

Expression of the result

$$Y \pm u_Y$$

uncertainty value - round up in all the cases

number of significant figures = 2

result Y - same number of decimal places as the uncertainty

$$\lambda = (632.8 \pm 1.3) \text{ nm}$$

$$T = (1.235 \pm 0.032) \text{ s}$$

$$E = (2.05 \pm 0.12) \cdot 10^{11} \text{ Pa}$$