

EMO-e at FTS CTU

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Subject website – <https://zolotarev.fd.cvut.cz/emoe> 

Lecture once a week – Thu 13:15

Practical laboratory exercises once in 14 days – Thu 15:00

- will start in the 1st week → **groups, timetable !**

Seminary exercise (**Thu 11:30**)

Supporting study literature

Lectures:

Halliday, D., Resnick, R., Walker, J.: Fundamentals of Physics (HRW)
pdf version at <http://gen.lib.rus.ec> 

Laboratory exercises:

subject website 

Assessment conditions

by 27th June 2025

compulsory practical education (fully passed)

successful delivery of all measurement reports (A - E)

Exam conditions

written part

4 problems – classification 0 – 2 points = **0 – 8 points total**

if 5/8 points are reached = oral exam

oral part – 2 topics from the list of topics (available on the website)

Pre-requisites of Physics

High school / grammar school physics level knowledge

physical quantities, units and basic laws
calculations without integration

Definition of vectors and scalars

direction of vector
components and magnitude of vector
addition and subtraction
dot product and cross product

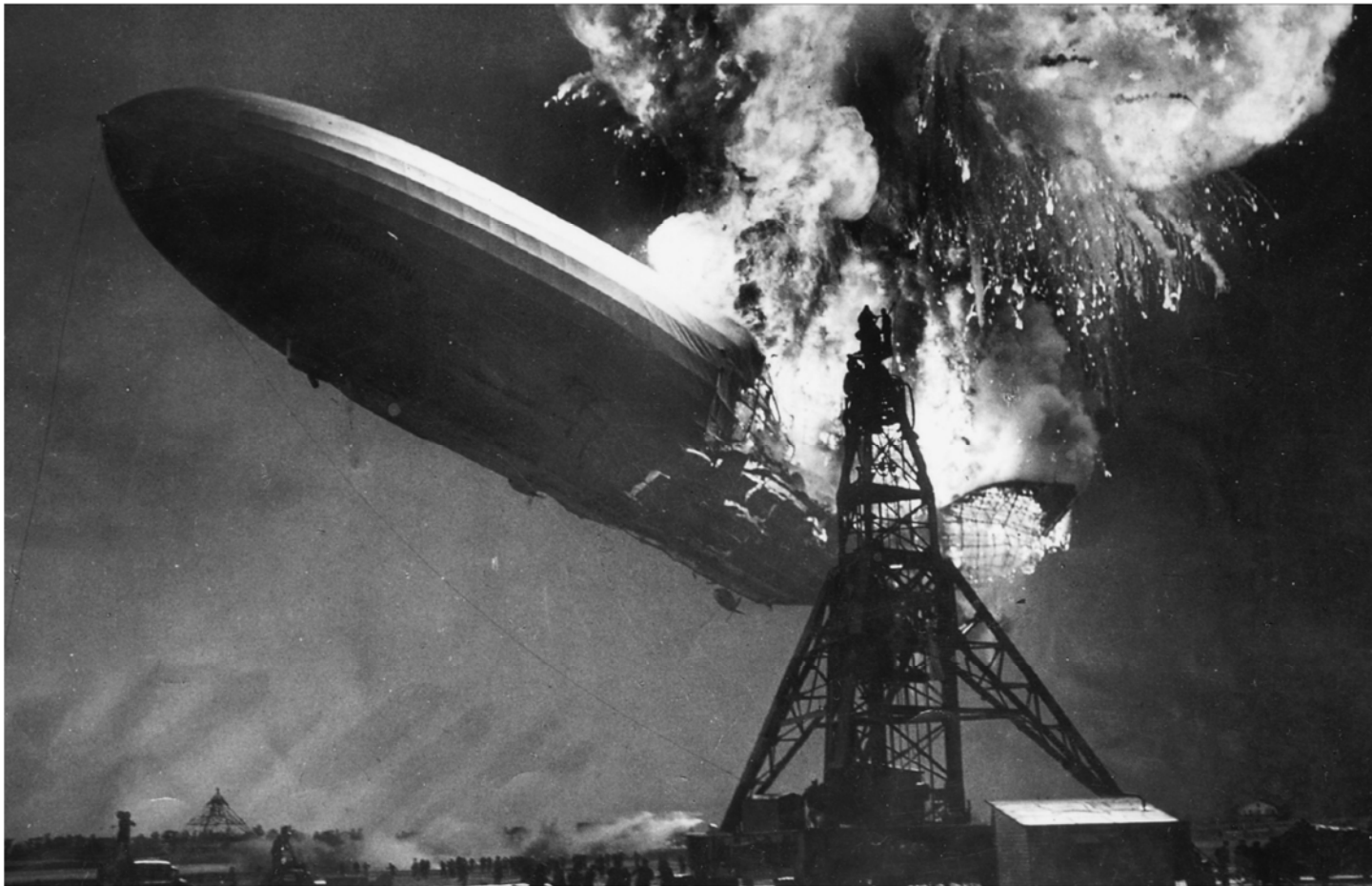
Basic knowledge of differential and integral calculus

one variable, multivariable

Basic physics knowledge

mechanics – motion equations, continuum, wave equations

Electric field



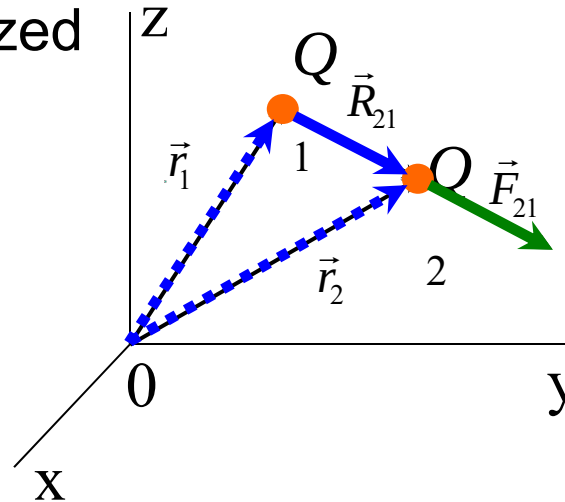
Coulomb's law

HRW: Ch21-22

electric charge, quantized

$$\vec{F}_{21} = k \frac{Q_1 Q_2}{R_{21}^2} \vec{r}_0$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$



$$k = \frac{1}{4\pi\epsilon_0}$$

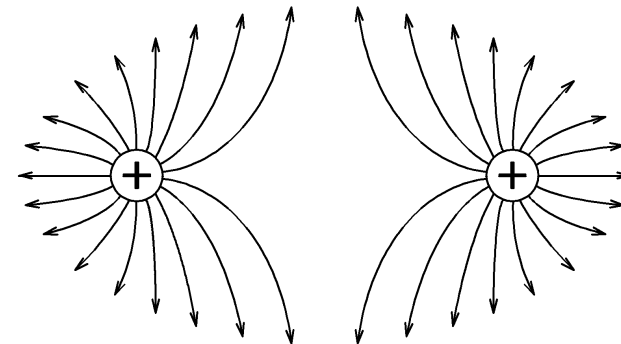
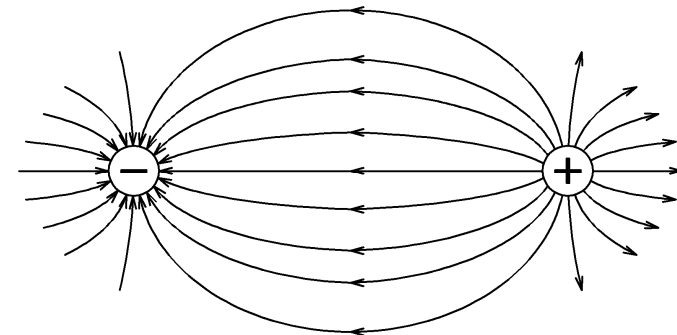
$$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ F} \cdot \text{m}^{-1}$$

Electric field (field intensity)

$$\vec{E} = \frac{\vec{F}}{Q}$$

field lines

$$\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\epsilon_0} \sum_i \frac{Q_i}{r_i^2} \vec{r}_{i0}$$



Charge density

linear $\tau = \lim_{\Delta l \rightarrow 0} \frac{\Delta Q}{\Delta l} = \frac{dQ}{dl}$

surface $\sigma = \frac{dQ}{dS}$

volumetric $\rho = \frac{dQ}{dV}$

Motion equation

$$\vec{F} = Q\vec{E}$$

$$m\vec{a} = Q\vec{E}$$

$$\vec{a} = \vec{E} \frac{Q}{m}$$

Electric dipole

$$\vec{p} = Q\vec{l} \quad \text{dipole moment}$$

$$\vec{M} = \vec{l} \times Q\vec{E} = \vec{l}Q \times \vec{E} = \vec{p} \times \vec{E}$$

$$dA = \vec{F} \cdot d\vec{s} = F_t ds = F_t r d\vartheta$$

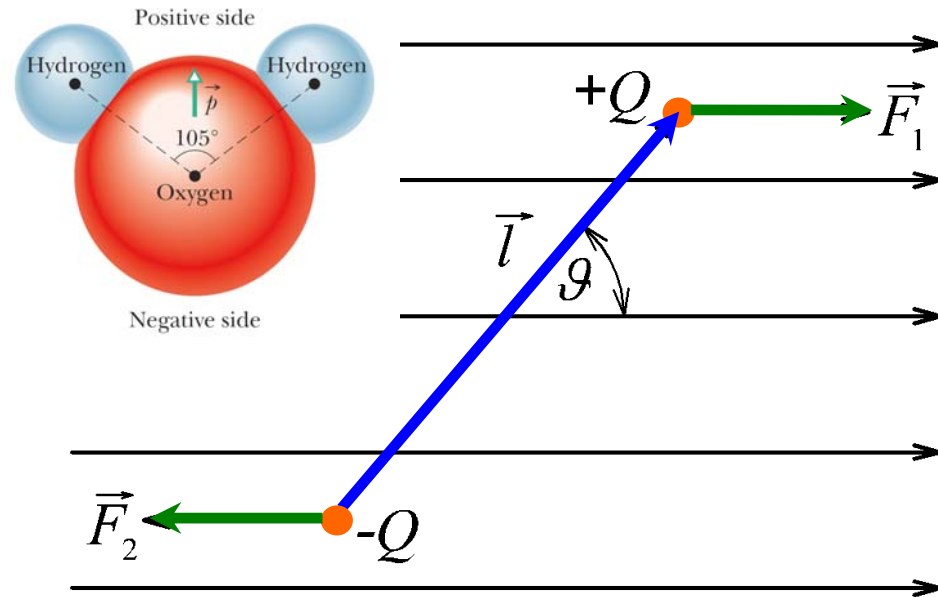
$$F_t r = M$$

$$dA = M d\vartheta$$

$$dW_p = M d\vartheta = pE \sin \vartheta d\vartheta$$

$$W_p = -pE \cos \vartheta + W_{p0}$$

$$W_p = -pE \cos \vartheta = -\vec{p} \cdot \vec{E}$$



$$\vartheta = \frac{\pi}{2} : \quad W_p = 0 \Rightarrow W_{p0} = 0$$

Gauss law

Gaussian surface – enclosed, flux ??

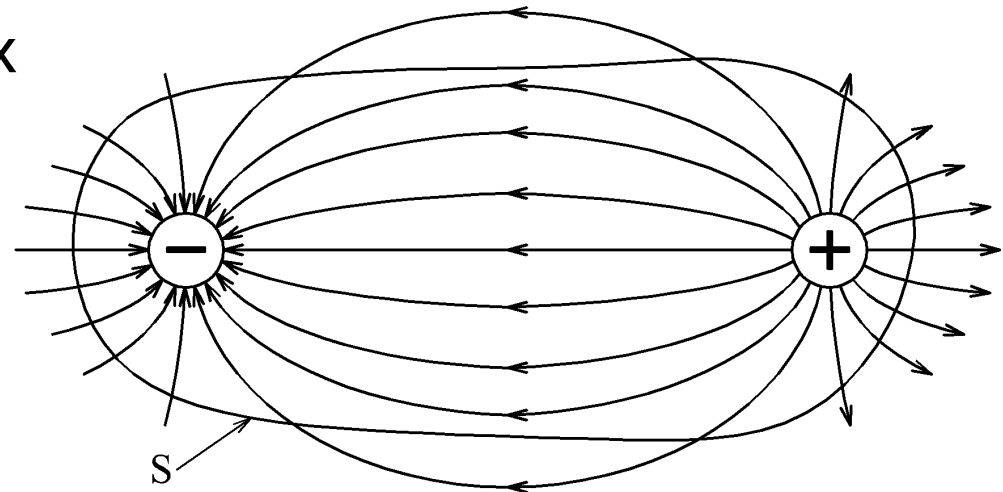
$$N(S) = \iint_S \vec{E} \cdot d\vec{S} \quad \text{electric flux}$$

$$N(S) = \oiint_S \vec{E} \cdot d\vec{S}$$

$$= \oiint_S \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} dS$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \oiint_S dS = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} 4\pi R^2 = \frac{1}{\epsilon_0} Q$$

$$N(S) = \oiint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \sum_i Q_i$$



$$\oiint_S \vec{E} \cdot d\vec{S} = \iiint_V \text{div } \vec{E} dV = \frac{1}{\epsilon_0} \iiint_V \rho dV$$

$$\text{div } \vec{E} = \frac{1}{\epsilon_0} \rho$$

Large uniformly charged non-conductive surface

$$\oiint_S \vec{E} \cdot d\vec{S} = \iint_{pl} \vec{E} \cdot d\vec{S} + \iint_{S_1} \vec{E} \cdot d\vec{S} + \iint_{S_2} \vec{E} \cdot d\vec{S}$$

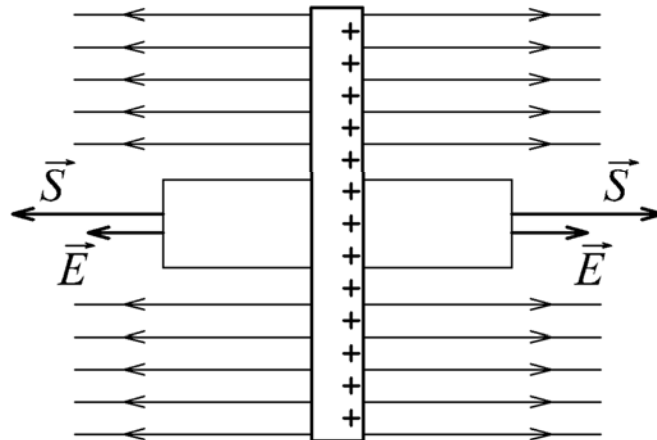
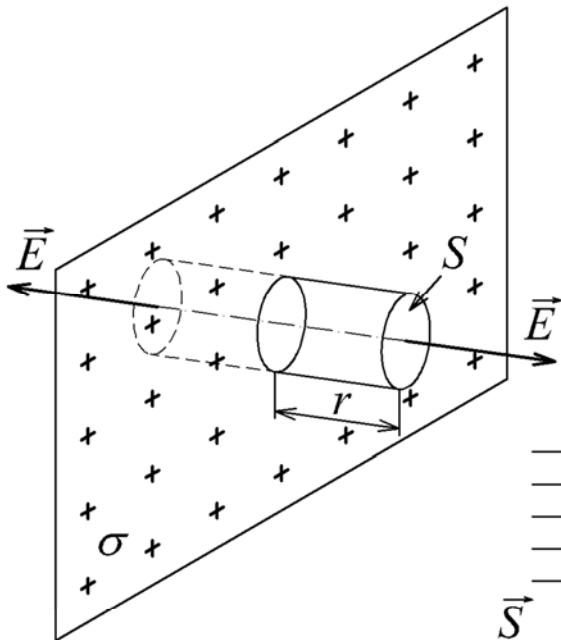
$$\iint_{pl} \vec{E} \cdot d\vec{S} = 0$$

$$\iint_S \vec{E} \cdot d\vec{S} = \iint_S E dS = E \iint_S dS$$

$$\iint_S \vec{E} \cdot d\vec{S} = EA$$

$$\oiint_S \vec{E} \cdot d\vec{S} = 2EA$$

$$Q = \sigma A \quad E = \frac{\sigma}{2\epsilon_0}$$

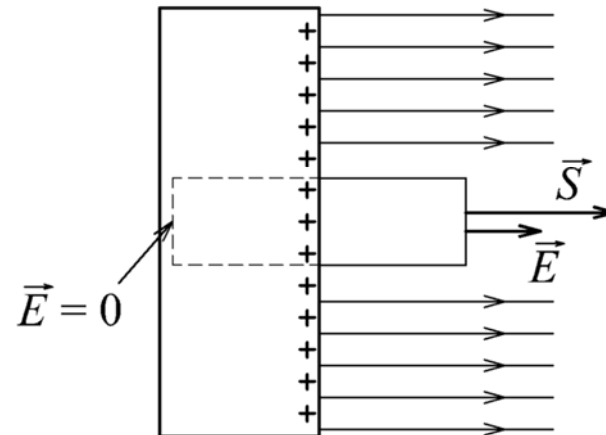
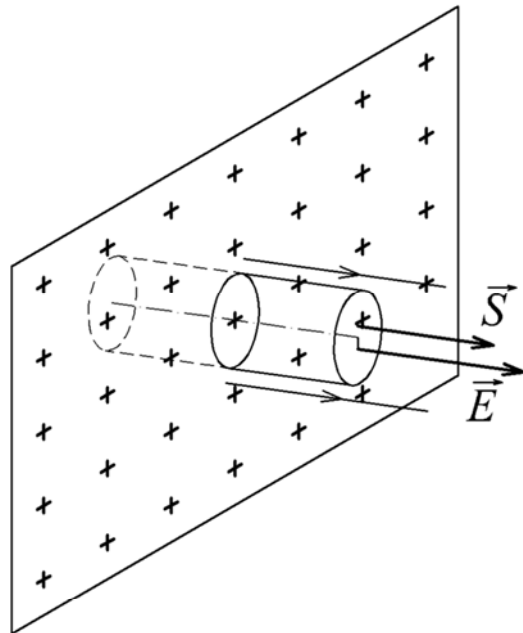


Large isolated conductor with positive charge

$$\oiint_S \vec{E} \cdot d\vec{S} = \iint_{pl} \vec{E} \cdot d\vec{S} + \iint_{S_i} \vec{E} \cdot d\vec{S} + \iint_{S_e} \vec{E} \cdot d\vec{S}$$

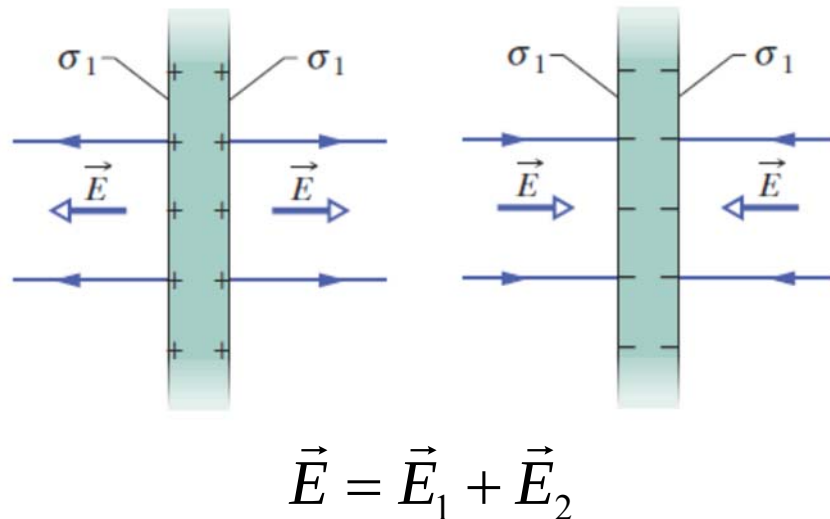
$$\iint_{S_e} \vec{E} \cdot d\vec{S} = \iint_{S_e} E dS = E \iint_{S_e} dS = EA$$

$$\oiint_S \vec{E} \cdot d\vec{S} = EA$$



$$Q = \sigma A \quad E = \frac{\sigma}{\epsilon_0}$$

Two conducting charged plates



oppositely charged

outside $E = E_1 - E_2 = 0$

inside $E = E_1 + E_2$

$$E = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2) = \frac{\sigma}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$

Electric Potential

$$dW_p = -Q\vec{E} \cdot d\vec{l} \quad d\phi = \frac{dW_p}{Q} = -\vec{E} \cdot d\vec{l}$$

$$\vec{E} = -\text{grad } \phi$$

conservative field

$$\oint_l \vec{E} \cdot d\vec{l} = 0$$

equipotential lines

$$\oint_l \vec{E} \cdot d\vec{l} = \iint_S \text{rot } \vec{E} \cdot d\vec{S} = 0$$

$$\text{rot } \vec{E} = 0$$