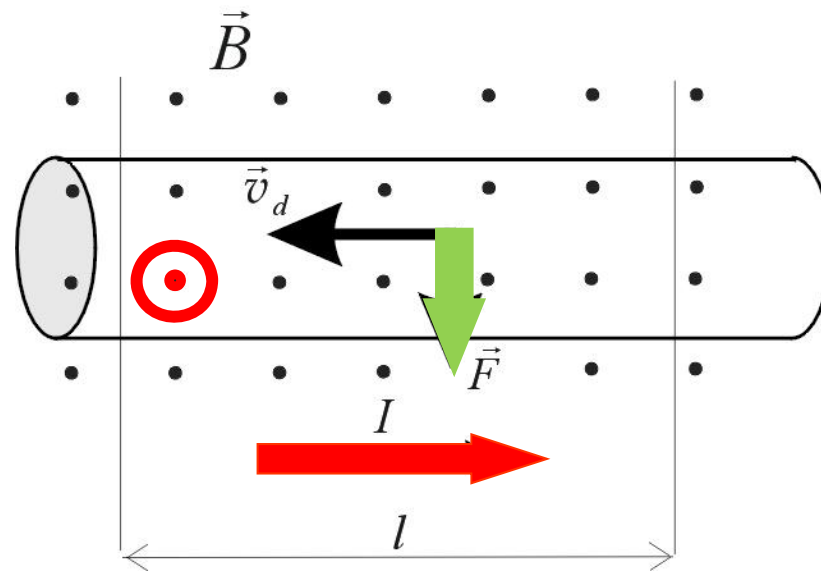


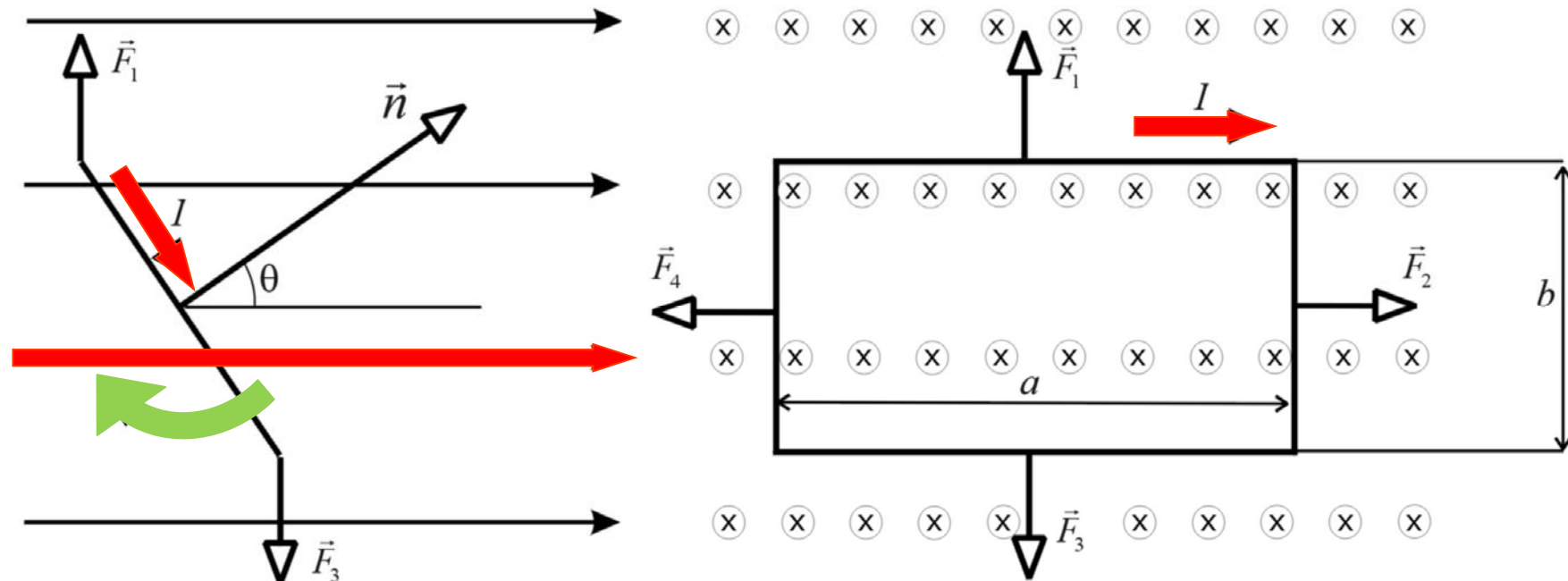
Force on a current-carrying wire

HRW: Ch28



$$\vec{F} = I\vec{l} \times \vec{B} \quad F = IlB \sin \varphi$$

Torque on a current loop



$$M = bF_1 \sin \theta$$

$$F_1 = IaB, (\sin \varphi = 1)$$

$$M = IabB \sin \theta$$

$$\vec{M} = I\vec{S} \times \vec{B}$$

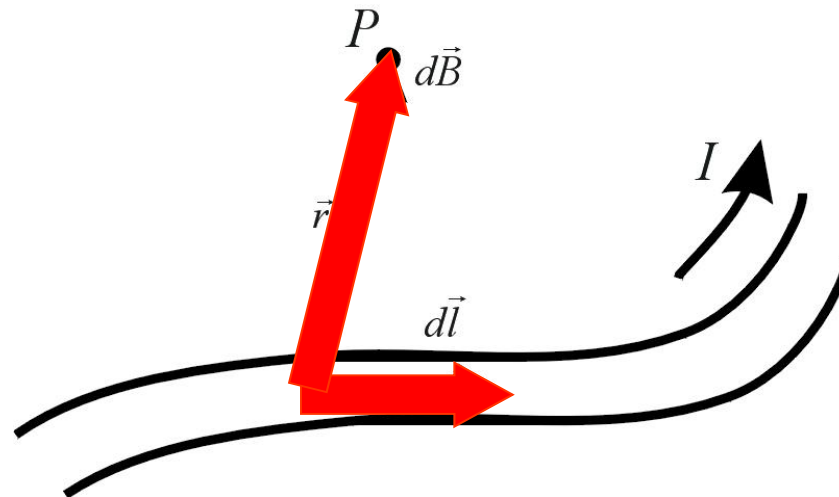
$$\vec{M} = NI\vec{S} \times \vec{B}$$

A torque acting on a current coil of N loops

$$\vec{m} = I\vec{S}$$

magnetic dipole moment

Magnetic field due to current: Biot-Savart law



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I (d\vec{l} \times \vec{r})}{r^3}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ T}\cdot\text{m}\cdot\text{A}^{-1}$$

permeability constant

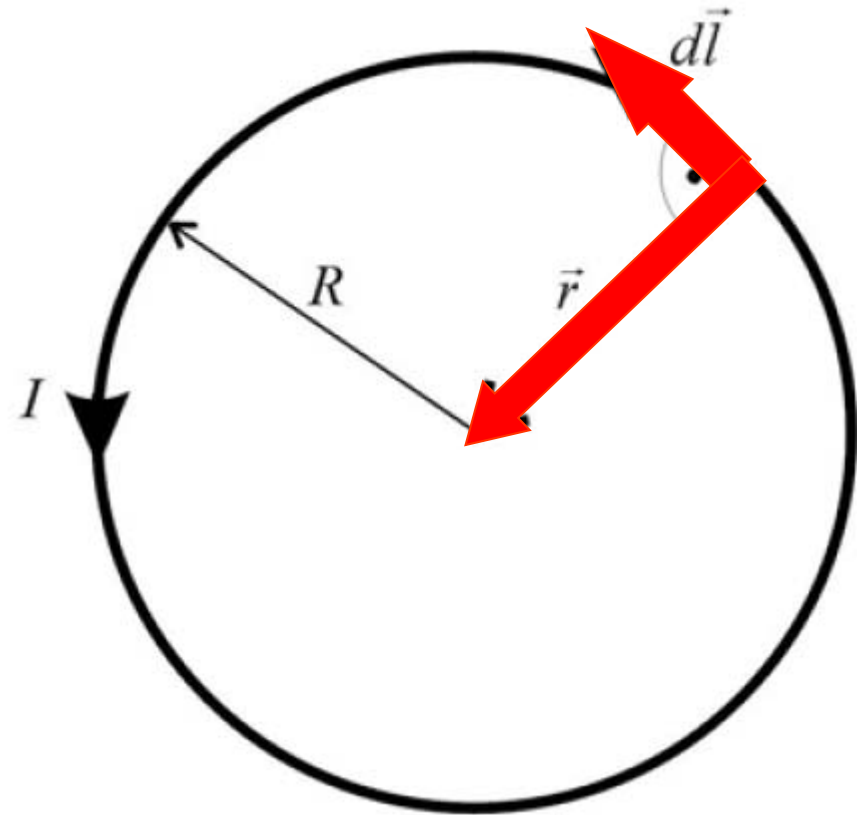
Magnetic field due to the circular current loop

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I (d\vec{l} \times \vec{r})}{r^3}$$

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{R^2} = \frac{\mu_0 I}{4\pi} \frac{dl}{R^2}$$

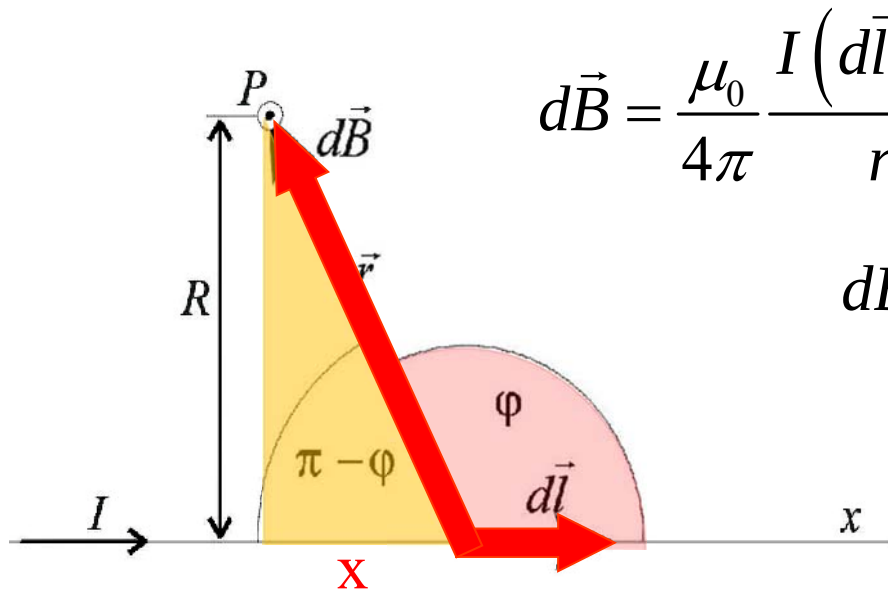
$$B = \int_0^{2\pi R} \frac{\mu_0 I}{4\pi} \frac{dl}{R^2} = \frac{\mu_0 I}{4\pi} \frac{1}{R^2} \int_0^{2\pi R} dl$$

$$B = \frac{\mu_0 I}{4\pi} \frac{1}{R^2} 2\pi R = \frac{\mu_0 I}{2R}$$



$$B = \frac{\mu_0 I}{2R}$$

Magnetic field due to a current in a long straight wire



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I (d\vec{l} \times \vec{r})}{r^3}$$

$$\vec{B} = \int d\vec{B} \quad B = \int dB$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dl r \sin \varphi}{r^3} = \frac{\mu_0}{4\pi} \frac{I dl \sin \varphi}{r^2}$$

$$dl = dx$$

$$r^2 = x^2 + R^2$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dx \sin \varphi}{x^2 + R^2}$$

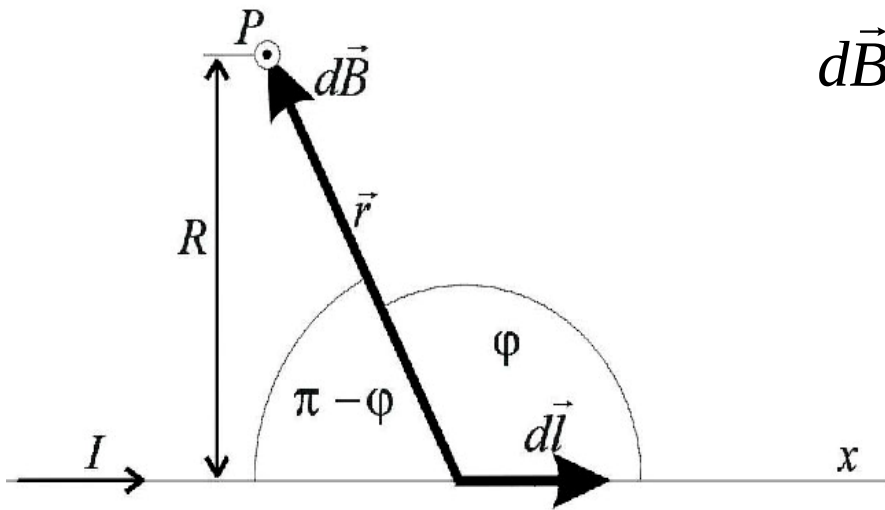
$$\sin \varphi = \sin(\pi - \varphi) = \frac{R}{(x^2 + R^2)^{\frac{1}{2}}}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I R d\varphi \sin \varphi}{\sin^2 \varphi (x^2 + R^2)}$$

$$\cotg(\varphi) = -\cotg(\pi - \varphi) = -\frac{x}{R}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I d\varphi \sin \varphi}{R}$$

$$-\frac{d\varphi}{\sin^2 \varphi} = -\frac{dx}{R}$$



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I (d\vec{l} \times \vec{r})}{r^3}$$

$$B = \int_0^\pi \frac{\mu_0}{4\pi} \frac{I \sin \varphi}{R} d\varphi = \frac{\mu_0 I}{4\pi R} [-\cos \varphi]_0^\pi = \frac{\mu_0 I}{2\pi R}$$

$$B = \frac{\mu_0 I}{2\pi R}$$

Magnetic field due to the circular current loop

$$B = \frac{\mu_0 I}{2R}$$

Magnetic field due to a current in a long straight wire

$$B = \frac{\mu_0 I}{2\pi R}$$

Force between two parallel currents

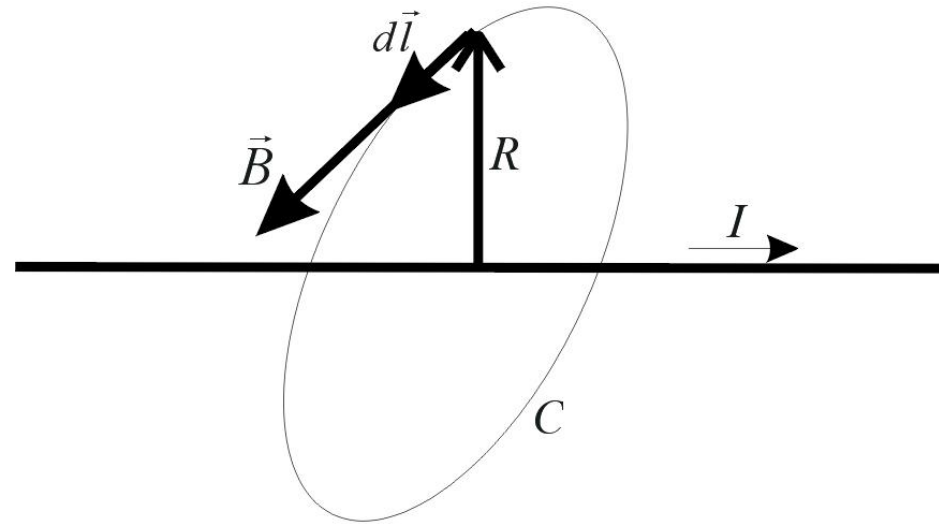
$$F = B_1 I_2 l = \frac{\mu_0 I_1 I_2}{2\pi R} l$$

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, and placed one metre apart in vacuum, would produce between these conductors a force equal to $2 \cdot 10^{-7}$ N per metre of length.

Ampère's law

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$\oint \vec{B} \cdot d\vec{l} \neq 0$$

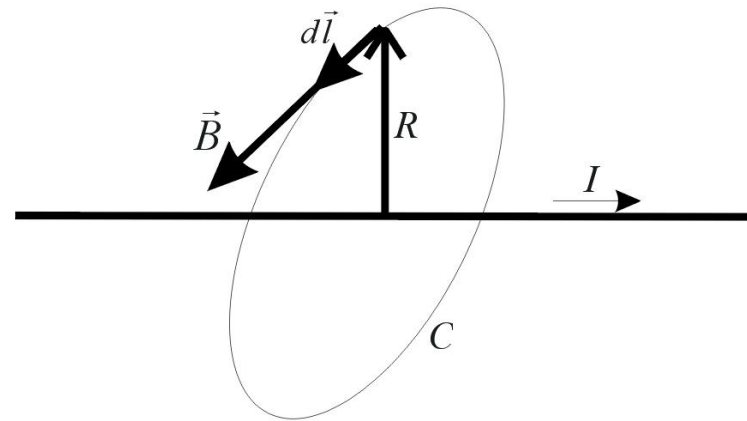


$$\oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = B 2\pi R = \frac{\mu_0 I}{2\pi R} 2\pi R = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi R}$$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \sum_i I_i$$

Ampère's law

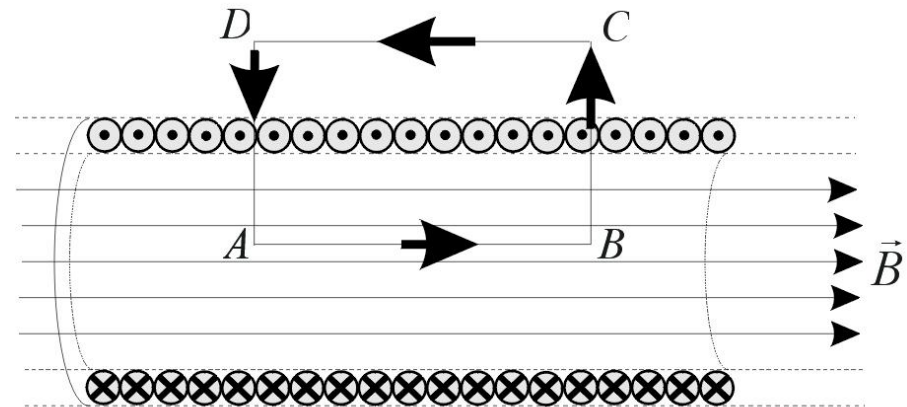


Magnetic field of an ideal solenoid

$$\oint_{ABCD} \vec{B} \cdot d\vec{l} = \int_{AB} \vec{B} \cdot d\vec{l} + \int_{BC} \vec{B} \cdot d\vec{l} + \int_{CD} \vec{B} \cdot d\vec{l} + \int_{DA} \vec{B} \cdot d\vec{l}$$

$$\vec{B} \perp d\vec{l} \Rightarrow \int_{BC} \vec{B} \cdot d\vec{l} = \int_{DA} \vec{B} \cdot d\vec{l} = 0$$

$$\oint_{ABCD} \vec{B} \cdot d\vec{l} = \oint_{AB} \vec{B} \cdot d\vec{l} = \mu_0 \sum_i I_i$$



$$Bl_1 = \mu_0 N_1 I$$

$$B = \frac{\mu_0 N_1 I}{l_1} = \frac{\mu_0 NI}{l}$$