Induction and Inductance

Magnetic flux

$$\Phi = \iint_S \vec{B} \cdot d\vec{S}$$

$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$

$$[\Phi] = \mathrm{m}^2.\mathrm{kg}.\,\mathrm{s}^{-2}.\,\mathrm{A}^{-1} = \mathrm{Wb}$$

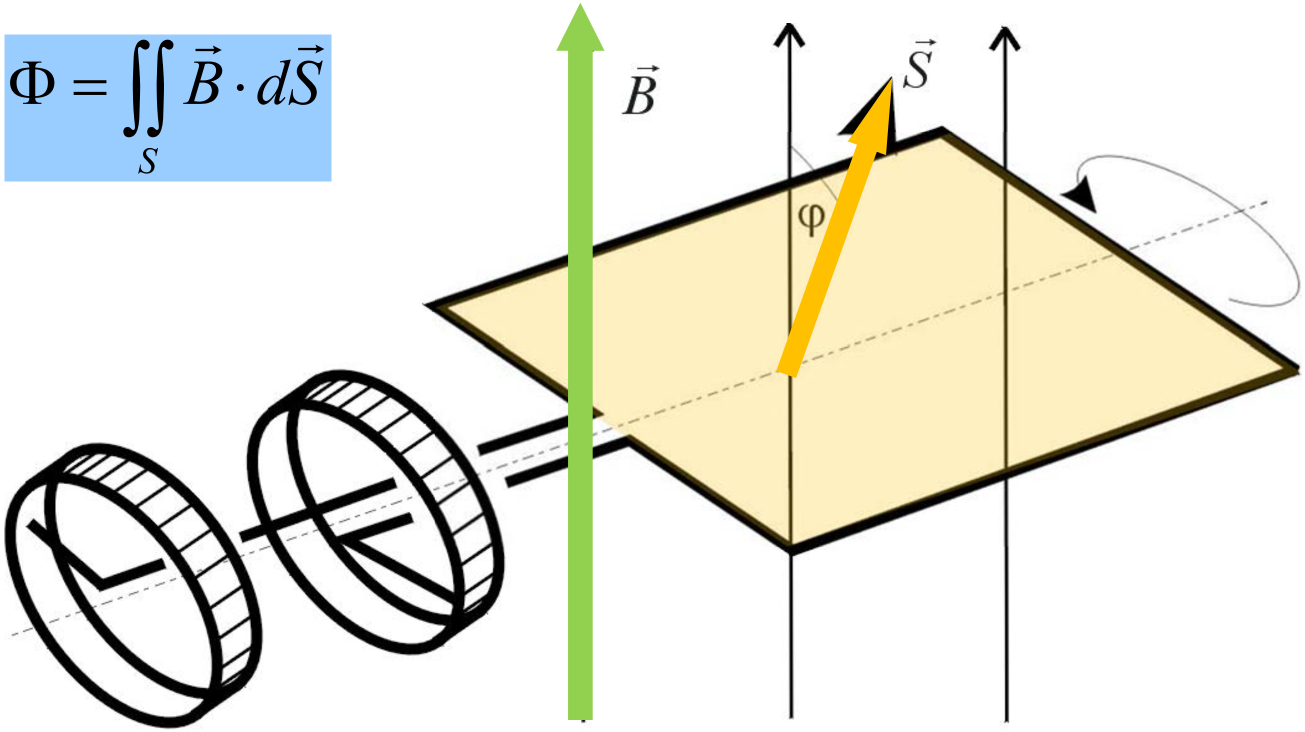
Faraday's law of induction

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

The magnitude of the emf ε induced in a conducting loop is equal to the rate at which the magnetic flux Φ through that loop changes with time.

Lenz's law = induced emf tends to oppose the flux change

$$\Phi = \iint_S \vec{B} \cdot d\vec{S}$$

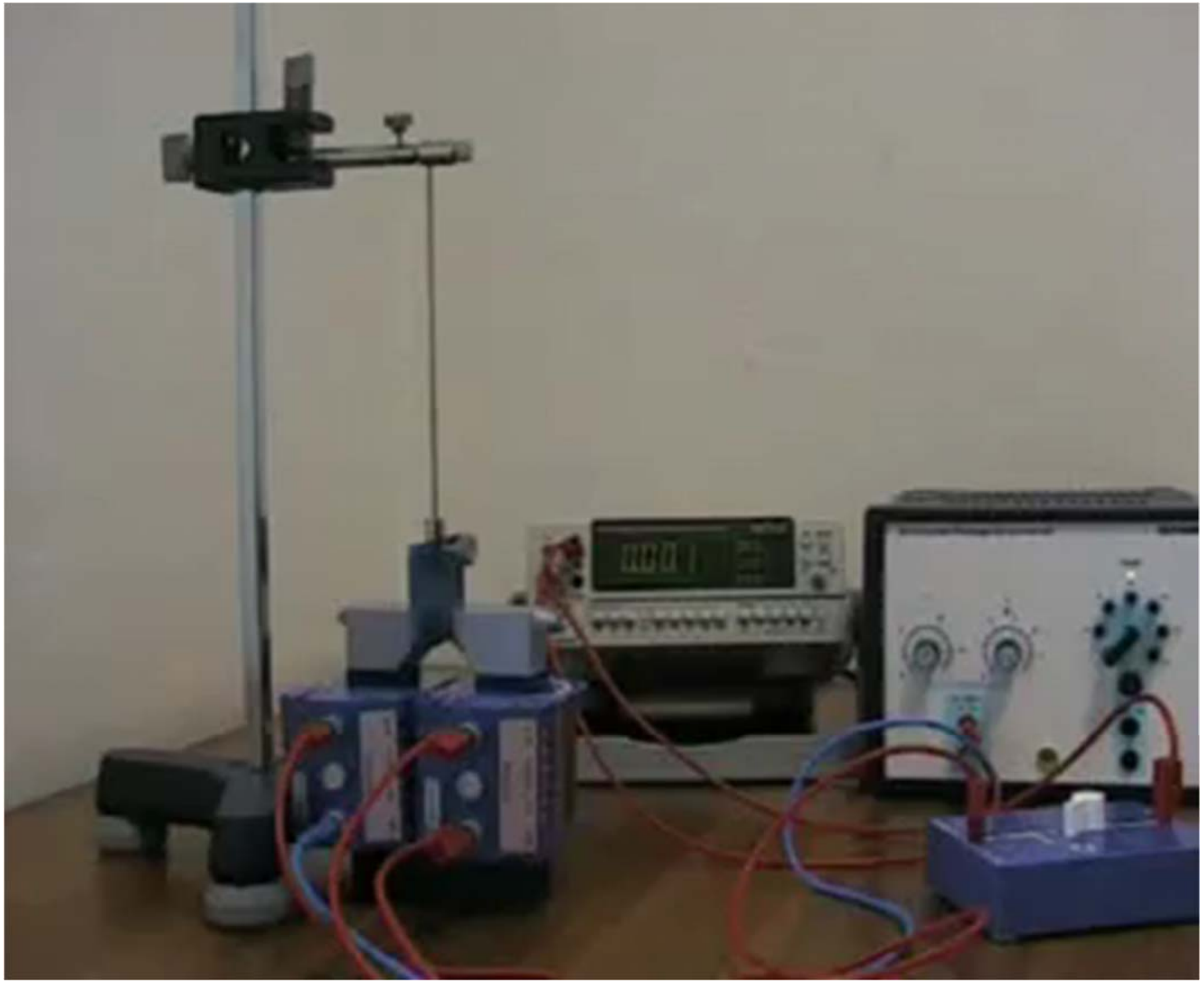


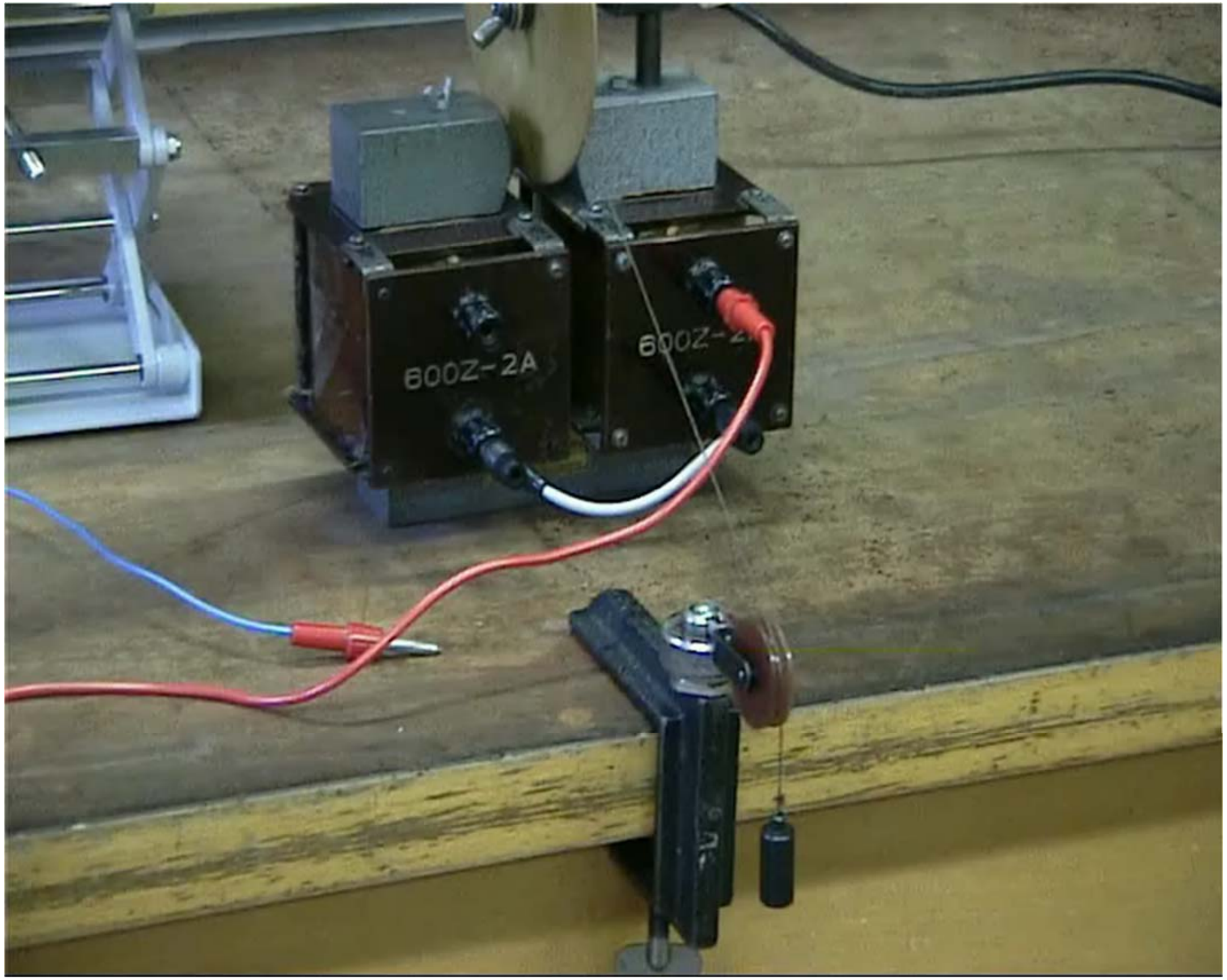
$$\Phi = BS\cos\varphi = BS\cos\omega t$$

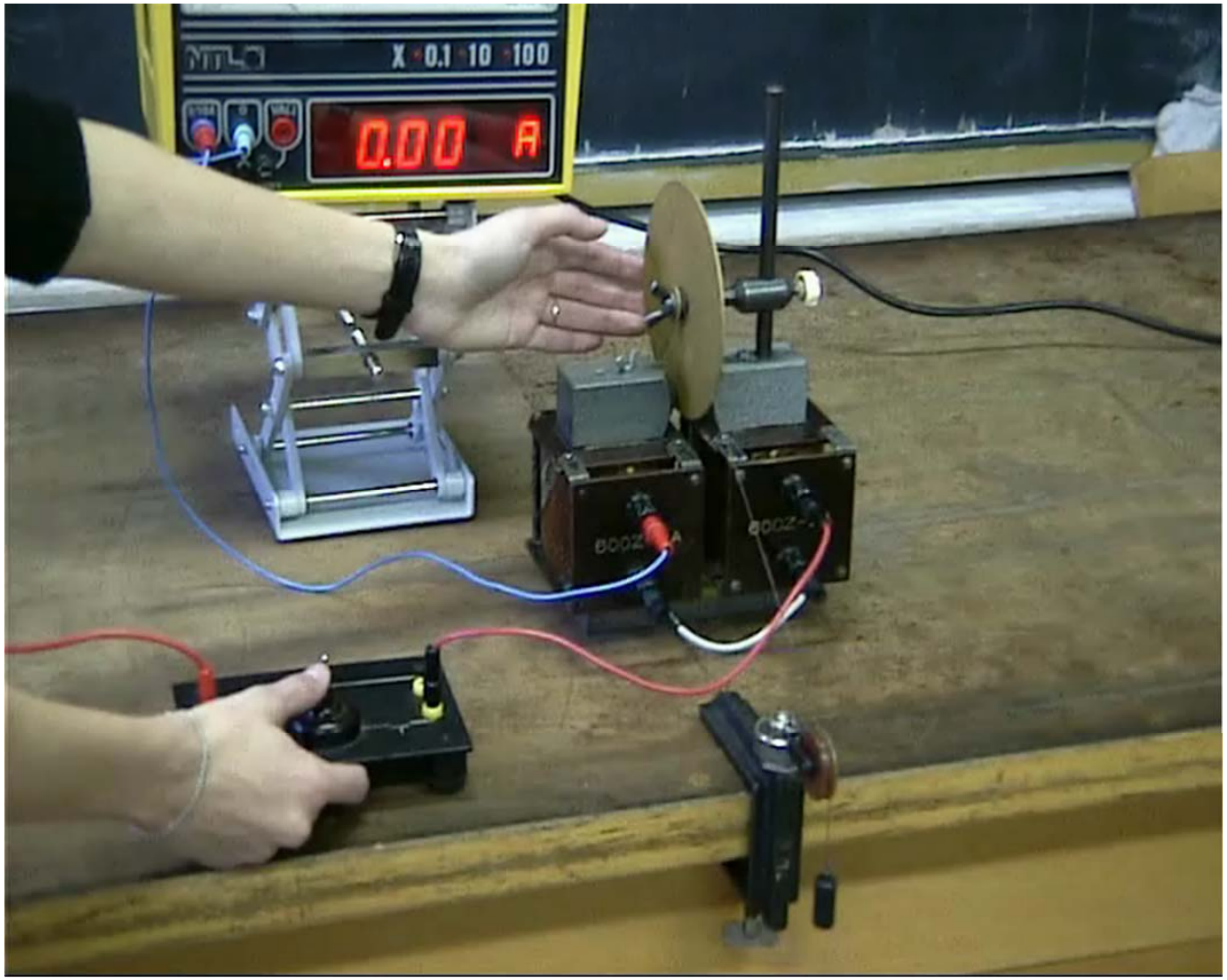
$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(BS\cos\omega t\right) = BS\omega\sin\omega t$$

eddy (Foucault's) currents

https://phet.colorado.edu/sims/cheerpj/faraday/latest/faraday.html?simulation=generator




600Z-2A

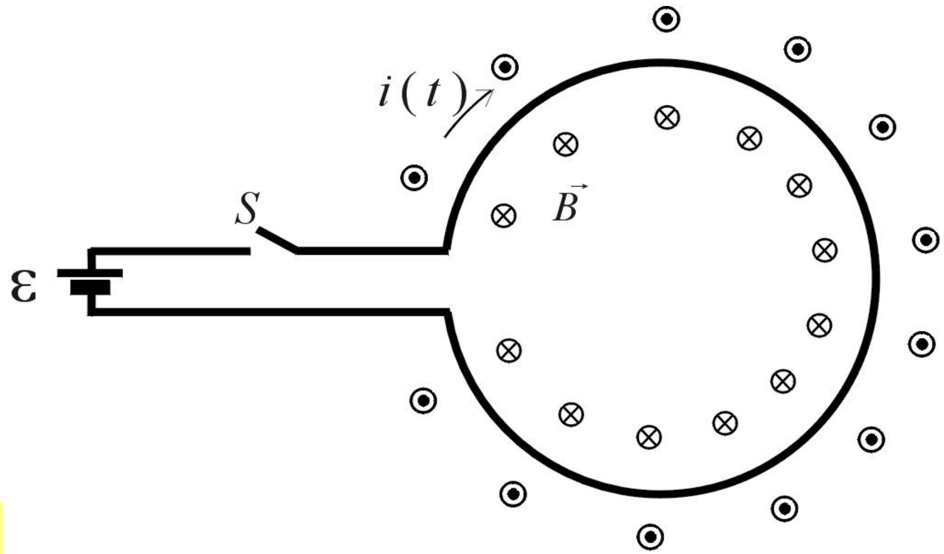

X 0.1 10 100
0.00 A

Inductors and Inductance

Self-induction

$$\Phi = Li$$

$$\mathcal{E} = -L\frac{di}{dt}$$



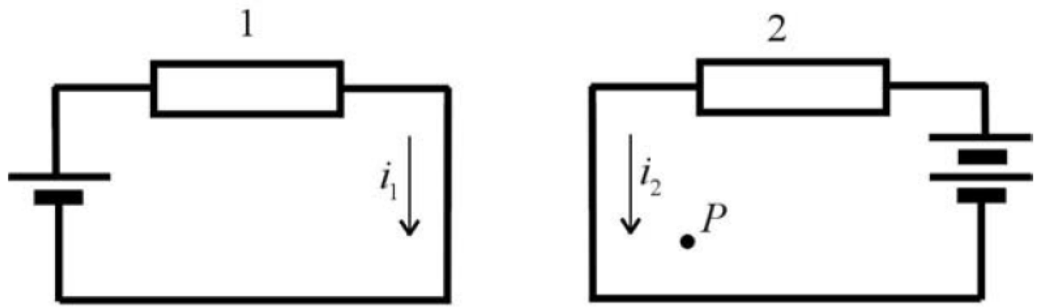
Self-induction of a solenoid

$$B = \mu_0 \frac{iN}{l}$$

$$\Phi = BNS$$

$$L = \mu_0 \frac{N^2 S}{l}$$

Mutual induction

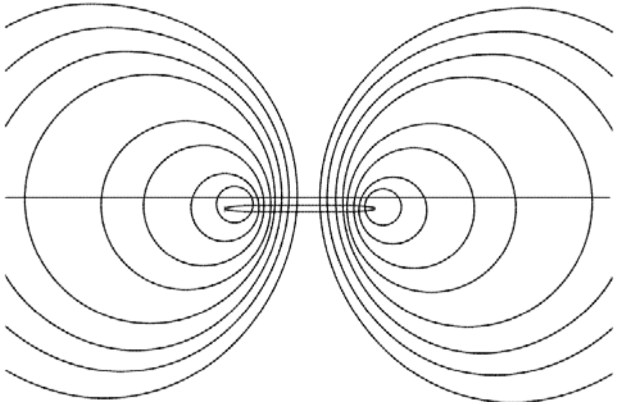


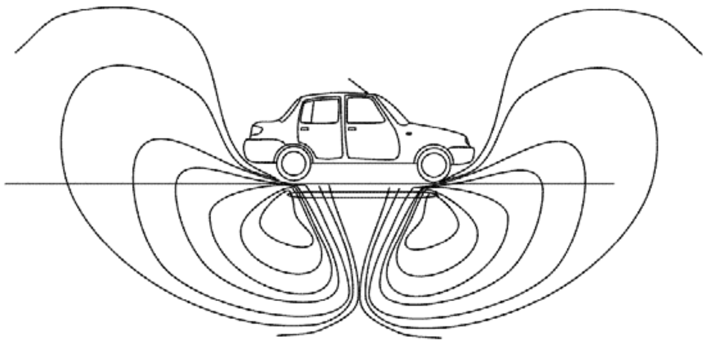
$$\Phi_2 = L_2 i_2 + M_{12} i_1$$

$$M_{12} = M_{21}$$

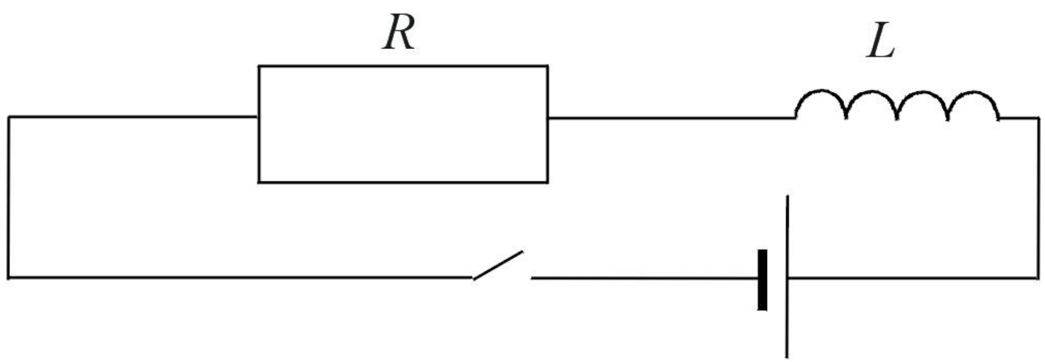
$$\Phi_1 = L_1 i_1 + M_{21} i_2$$

$$\mathcal{E}_1 = -\frac{d\Phi_1}{dt} = -L_1 \frac{di_1}{dt} - M_{12} \frac{di_2}{dt}$$





RL circuit



$$Ri = \mathcal{E}_0 - L\frac{di}{dt}$$

$$L\frac{di}{dt} + Ri = \mathcal{E}_0$$

$$Li\frac{di}{dt} + Ri^2 = \mathcal{E}_0 i$$

$$p = ui = Li\frac{di}{dt}$$

$$dW = p\,dt = Li\,di$$

$$W = \int dW = \int_0^{i_m} Li\,di = \frac{1}{2}Li_m^2$$

$$W_m = \frac{1}{2}Li^2$$

Energy of magnetic field

Energy stored in inductor (solenoid) $$W_m = \frac{1}{2} L i^2$$

$$L = \mu_0 \frac{N^2 S}{l}$$

$$W_m = \frac{1}{2} \mu_0 N^2 \frac{S}{l} i^2$$

$$B = \mu_0 \frac{N i}{l}$$

$$W_m = \frac{1}{2} \frac{1}{\mu_0} B^2 S l$$

Energy density of a magnetic field $$w_m = \frac{W_m}{V} = \frac{1}{2} \frac{1}{\mu_0} B^2$$

Energy stored in a magnetic field $$W_m = \int_V w_m \, dV$$

Alternating current

Periodic function (generation of emf by rotation) – harmonic current

$$i(t) = I_0 \sin(\omega t + \varphi_0)$$

emf function (phase shift φ)

$$u(t) = U_0 \sin(\omega t + \varphi_0 + \varphi)$$

driving angular frequency, phase shift φ (load dependent)

Effective value (RMS)

$$I_{ef} = I_0 \sqrt{\frac{1}{T}\int_0^T \sin^2(t)\mathrm{dt}} = I_0 \frac{1}{\sqrt{2}} \qquad \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

Power in AC circuits $P = UI\cos\varphi$ power factor **cos** $\boldsymbol{\varphi}$