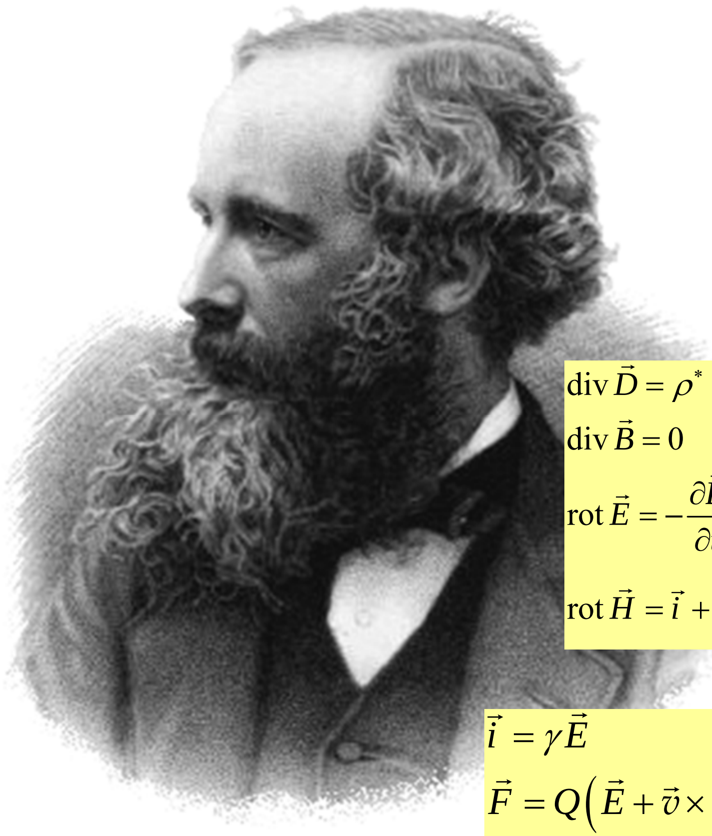
HRW: Ch32

# Maxwell’s Equations



$$\mathrm{div}\,\vec{D} = \rho^{*}$$

$$\mathrm{div}\,\vec{B} = 0$$

$$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\mathrm{rot}\,\vec{H} = \vec{i} + \frac{\partial \vec{D}}{\partial t}$$

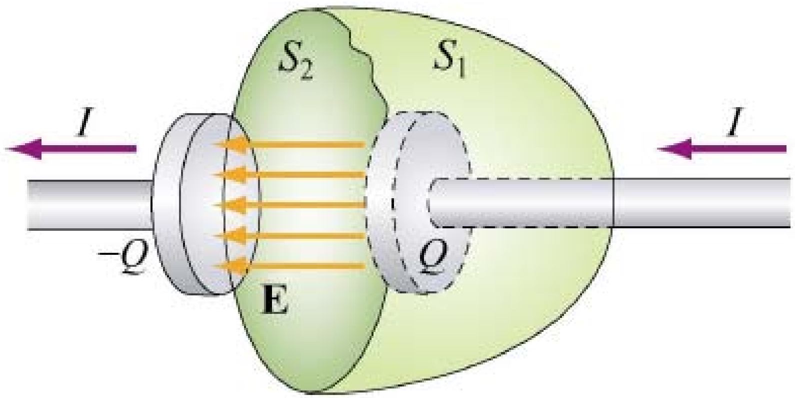
$$\vec{i} = \gamma \vec{E}$$

$$\vec{F} = Q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}, \quad \vec{B} = \mu_0 \mu_r \vec{H}$$

# Displacement current

$$\oint_{C(S)} \vec{B} \cdot d\vec{l} = \mu \sum I = \mu \iint_S \vec{i} \cdot d\vec{S}$$




$$\oint_{C(S)} \vec{B} \cdot d\vec{l} = \mu \left( \sum I + \frac{d}{dt} \iint_S \vec{D} \cdot d\vec{S} \right) = \mu \iint_S \left( \vec{i} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

Maxwell's law of induction

$$\oint_{C(S)} \vec{H} \cdot d\vec{l} = \iint_S \left( \vec{i} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

Ampére-Maxwell law

# Maxwell's Equations

$$\oiint_S \vec{D} \cdot d\vec{S} = Q^*$$

Gauss' law of electricity

$$\operatorname{div} \vec{D} = \rho^*$$

$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$

Gauss' law of magnetism

$$\operatorname{div} \vec{B} = 0$$

$$\oint_{C(S)} \vec{E} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Faraday's law

$$\operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_{C(S)} \vec{H} \cdot d\vec{l} = \iint_S \left( \vec{i} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

Ampere-Maxwell's law

$$\operatorname{rot} \vec{H} = \vec{i} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{i} = \gamma \vec{E}$$

Ohm's law

$$\vec{F} = Q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Lorentz force

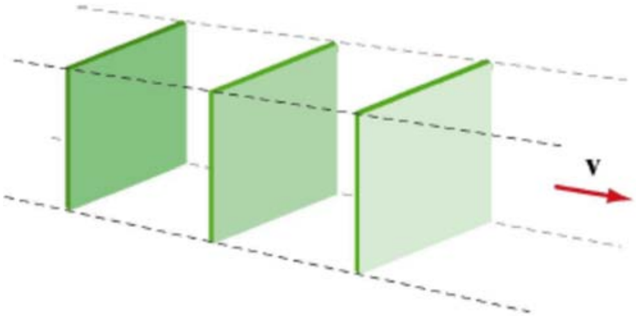
$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}, \quad \vec{B} = \mu_0 \mu_r \vec{H}$$

# Electromagnetic waves

a wave travelling in empty space $\rho^* = 0,\ \vec{i} = 0,\ \varepsilon_r = \mu_r = 1$

$$\Delta\vec{E} = \varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \triangle u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \qquad \Delta\vec{B} = \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$c^2 = \frac{1}{\varepsilon_0\mu_0} \qquad v^2 = \frac{1}{\varepsilon_0\varepsilon_r\mu_0\mu_r}$$

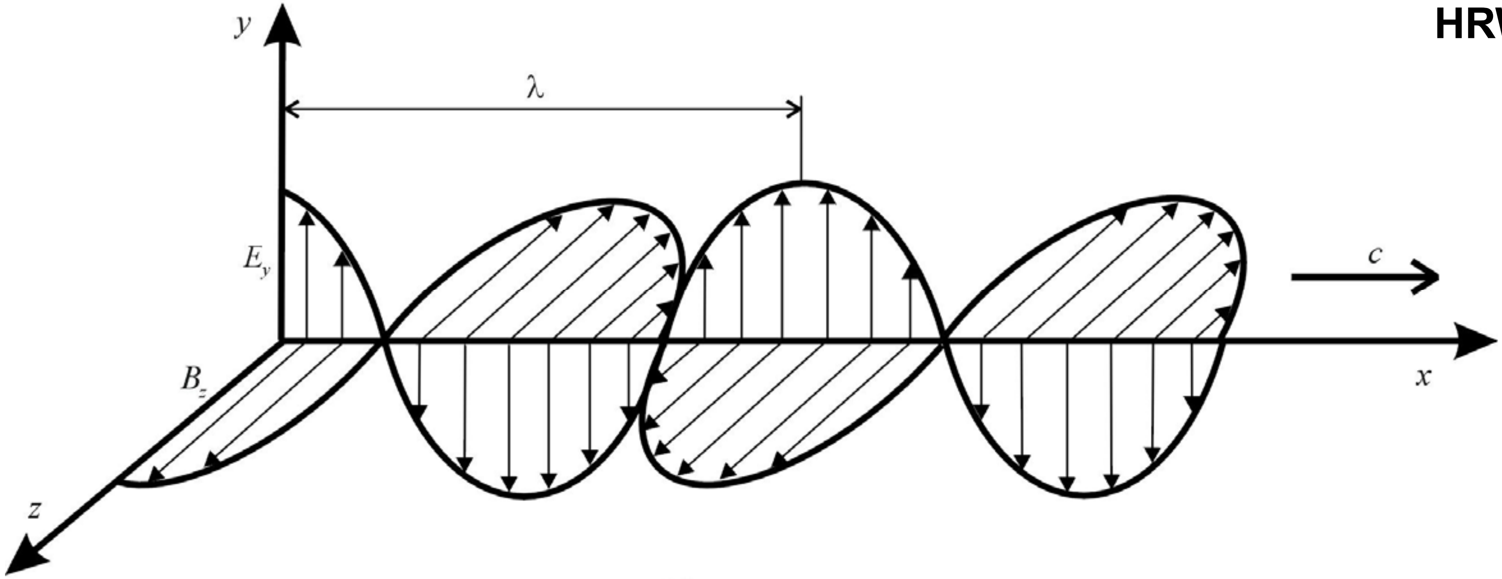



$$\frac{\partial E_x}{\partial t} = \frac{\partial B_x}{\partial t} = 0$$

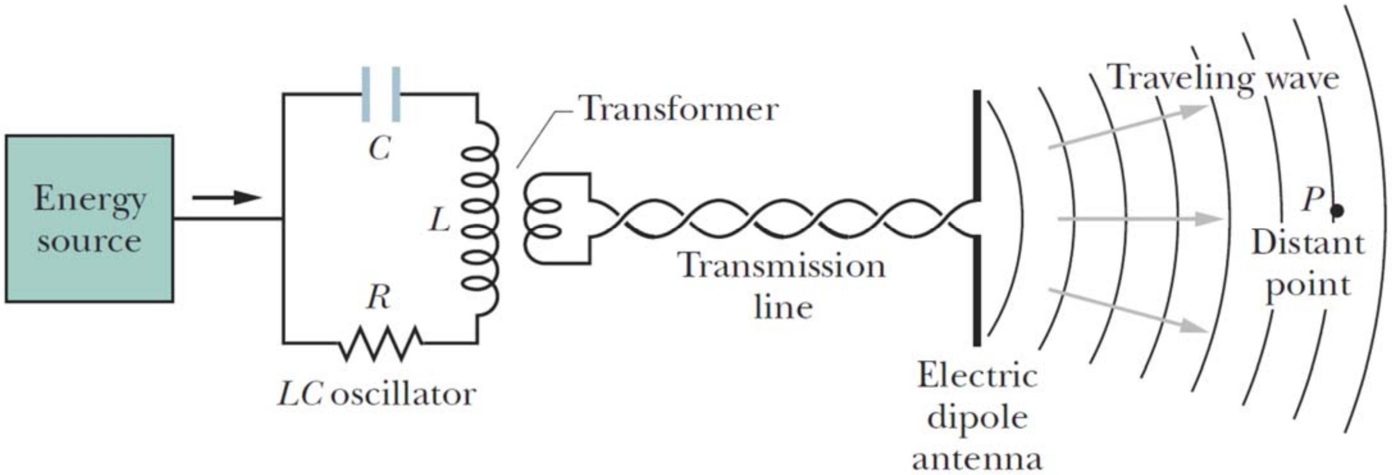
transverse wave

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} = 0 \Rightarrow B_z = B_z(t) \Rightarrow E \perp \vec{B}$$

$$B_z = \sqrt{\varepsilon_0\mu_0} E_0 \sin(kx - \omega t) = B_0 \sin(kx - \omega t) \qquad \frac{E_y}{B_z} = \frac{E_0}{B_0} = \frac{E}{B} = c$$



y
λ
$E_y$
c
$B_z$
x
z

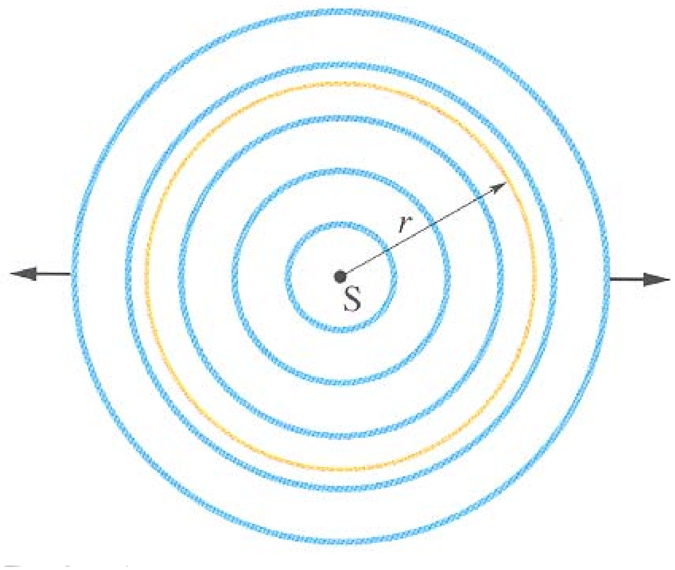
Energy source
C
Transformer
L
R
LC oscillator
Transmission line
Electric dipole antenna
Traveling wave
P
Distant point

# Energy transport and Poynting vector

density of power $S = \frac{1}{A}\frac{dW}{dt}$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Poynting vector




Electromagnetic wave in vacuum

$$S = \frac{1}{\mu_0} E_0 B_0 \sin^2\left(kx - \omega t\right)$$

$$\overline{S} = \overline{I} = \frac{1}{2\mu_0} E_0 B_0 = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2\mu_0} c B_0^2$$

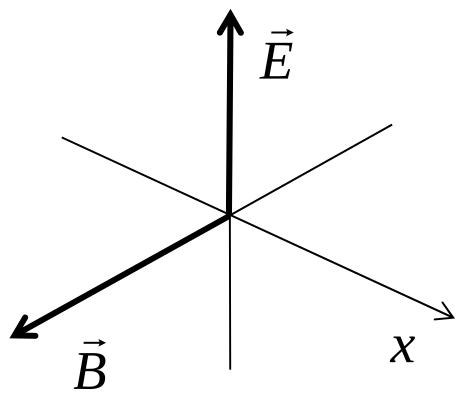
## Electromagnetic wave: density of energy

$$w = w_e + w_m = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

$$\overline{I} = c\overline{w} = \overline{S}$$

$$w = \frac{1}{2}\left(\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B}\right)$$

## momentum of electromagnetic wave




$$F = evB$$

$$\overline{F} = \frac{e\overline{vE}}{c} = \frac{vF_e}{c} = \frac{1}{c}\frac{dW}{dt}$$

$$\overline{F}dt = \frac{dW}{c}$$

$$p = \frac{I}{c} = \frac{S}{c}$$

radiation pressure $p_{rad}$

$$\frac{S}{c} < p_{rad} < \frac{2S}{c}$$

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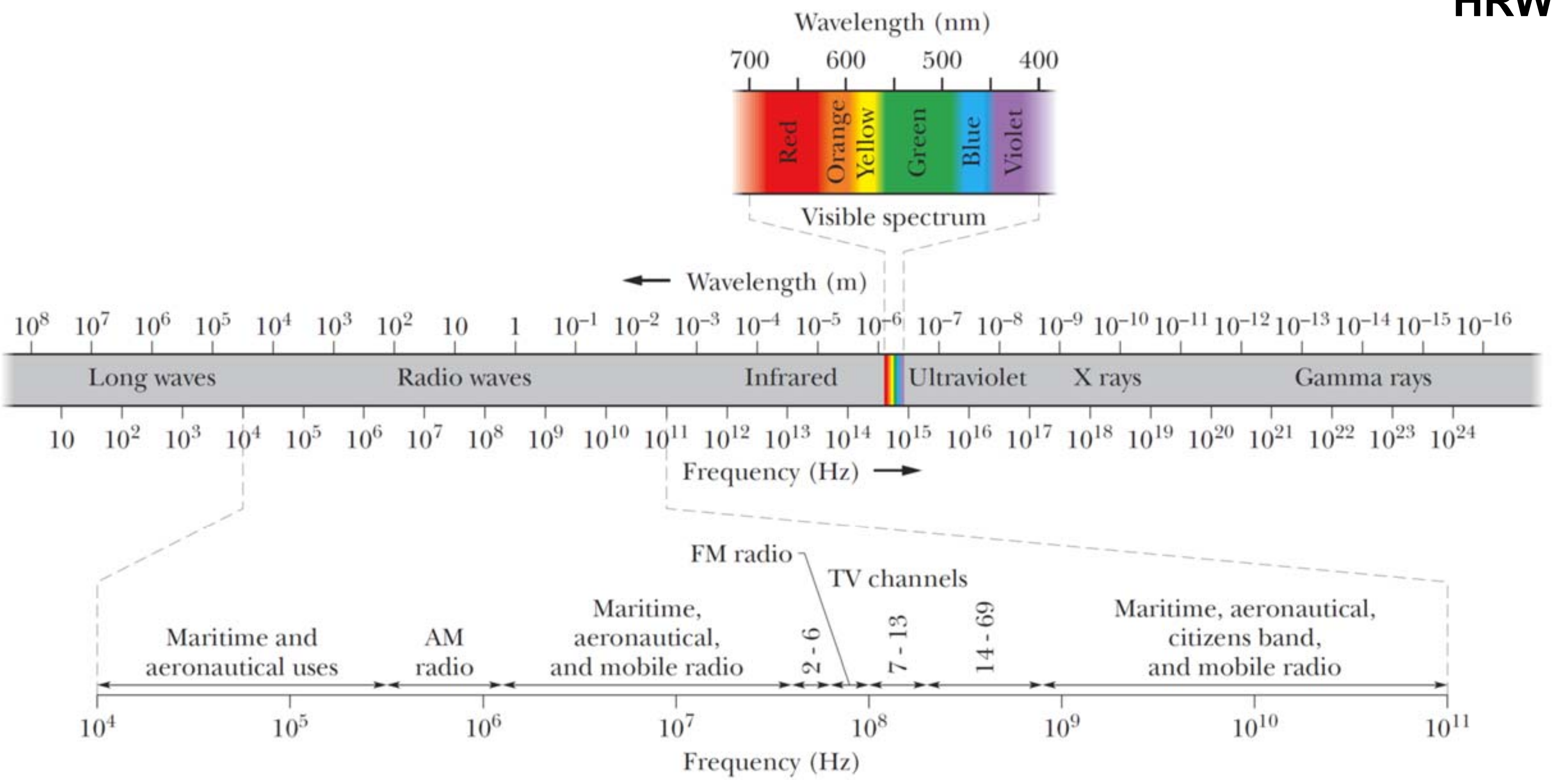
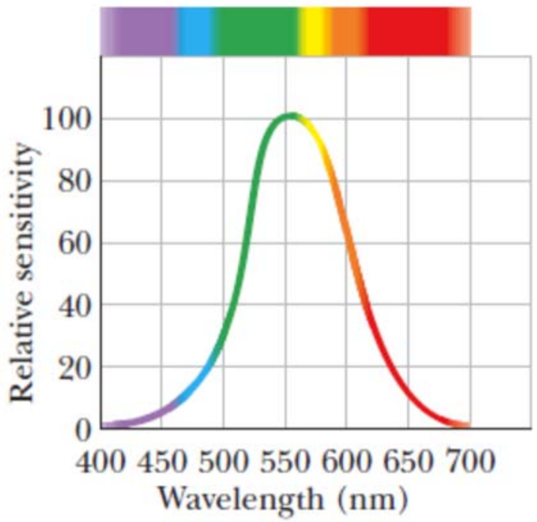

Wavelength (nm)
700 600 500 400
Red
Orange
Yellow
Green
Blue
Violet
Visible spectrum
Wavelength (m)
$10^8$ $10^7$ $10^6$ $10^5$ $10^4$ $10^3$ $10^2$ 10 1 $10^{-1}$ $10^{-2}$ $10^{-3}$ $10^{-4}$ $10^{-5}$ $10^{-6}$ $10^{-7}$ $10^{-8}$ $10^{-9}$ $10^{-10}$ $10^{-11}$ $10^{-12}$ $10^{-13}$ $10^{-14}$ $10^{-15}$ $10^{-16}$
Long waves
Radio waves
Infrared
Ultraviolet
X rays
Gamma rays
10 $10^2$ $10^3$ $10^4$ $10^5$ $10^6$ $10^7$ $10^8$ $10^9$ $10^{10}$ $10^{11}$ $10^{12}$ $10^{13}$ $10^{14}$ $10^{15}$ $10^{16}$ $10^{17}$ $10^{18}$ $10^{19}$ $10^{20}$ $10^{21}$ $10^{22}$ $10^{23}$ $10^{24}$
Frequency (Hz)
FM radio
TV channels
Maritime and aeronautical uses
AM radio
Maritime, aeronautical, and mobile radio
2 - 6
7 - 13
14 - 69
Maritime, aeronautical, citizens band, and mobile radio
$10^4$ $10^5$ $10^6$ $10^7$ $10^8$ $10^9$ $10^{10}$ $10^{11}$
Frequency (Hz)


Relative sensitivity
0 20 40 60 80 100
400 450 500 550 600 650 700
Wavelength (nm)