Photon - Quantum of Light

Thermal radiation

Radiant flux $\Phi_e = \frac{dW}{dt}$ Power emitted, reflected, transmitted, absorbed

Radiant exitance $M_e = \frac{d\Phi_e}{dS}$, $[M_e] = \mathrm{W} \cdot \mathrm{m}^{-2}$

Stefan-Boltzmann law $M_e = \varepsilon \sigma T^4$

$$\sigma = 5{,}67 \cdot 10^{-8}\,\mathrm{W} \cdot \mathrm{m}^{-2} \cdot \mathrm{K}^{-4}$$

emisivity $\varepsilon = \frac{M_{e,T}}{M_{e,T_{\alpha=1}}}$

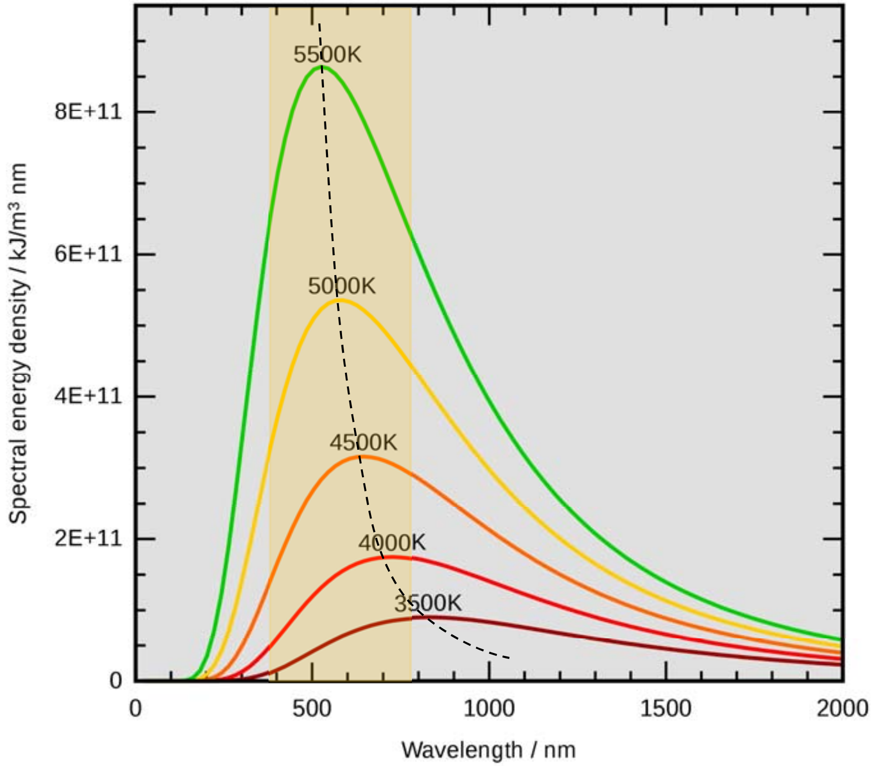
Black body – all electromagnetic radiation is absorbed

level of radiation equals to absorption ($\varepsilon = \alpha = 1$)

$$M_e = \sigma T^4$$

Spectral exitance

$$M_{e\lambda} = \frac{dM_e}{d\lambda} \quad , \quad [M_{e\lambda}] = \mathrm{W} \cdot \mathrm{m}^{-3}$$



$$\lambda^* T = b$$

Wien's displacement law

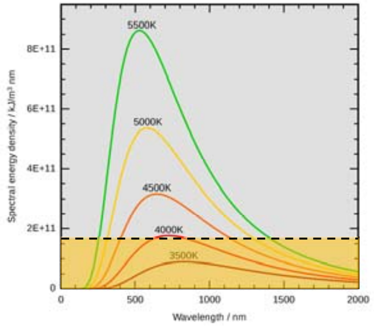
$$b = 2{,}8978 \cdot 10^{-3}\,\mathrm{m} \cdot \mathrm{K}$$

Thermal radiation laws - summary

Stefan-Boltzmann law

$$M_e = \varepsilon \sigma T^4$$

[W.m^{-2}]



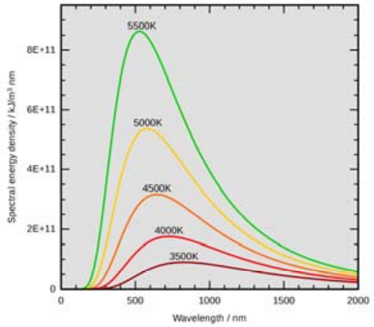
integration

$\lambda \in (-\infty;\infty)$

Planck's law (spectral density function)

$$M_{e\lambda} = \frac{2\pi h c^2}{\lambda^5 \left(e^{\frac{hc}{\lambda k T}} - 1\right)}$$

[W.m^{-3}]



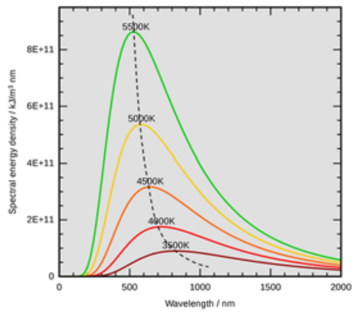
derivation

dλ

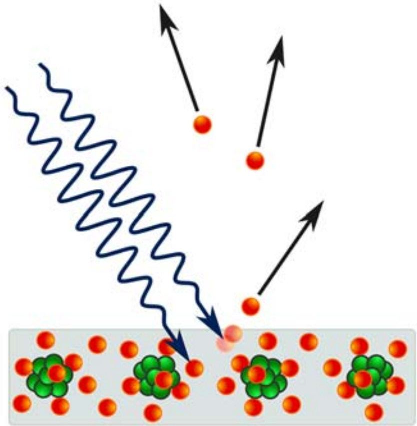
function extrema

Wien's displacement law (position of maximum)

$$\lambda^* T = b$$

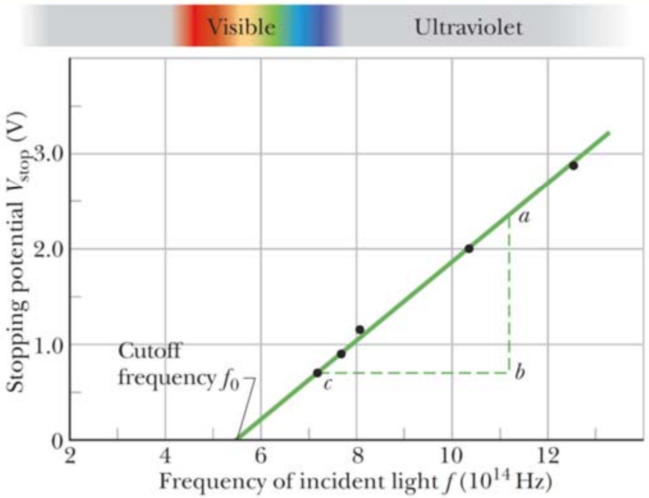


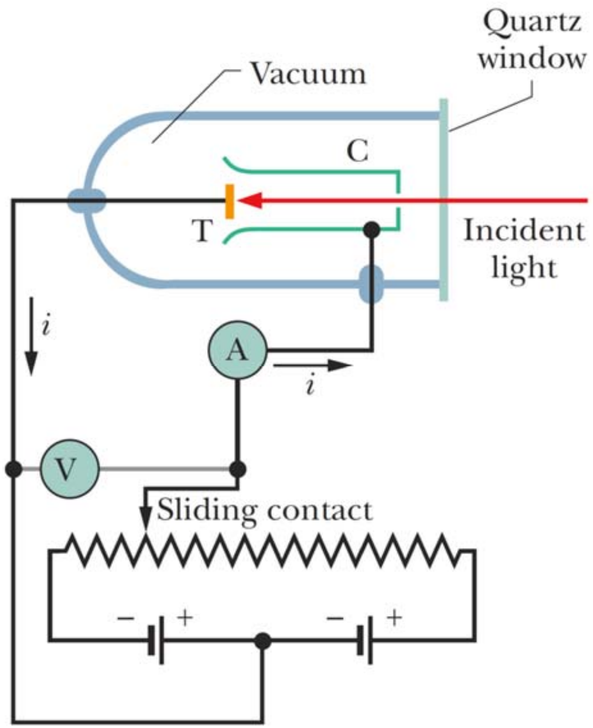
Photoelectric effect



Electrons can escape only if the light frequency exceeds a certain value.

The escaping electron's kinetic energy is greater for a greater light frequency.



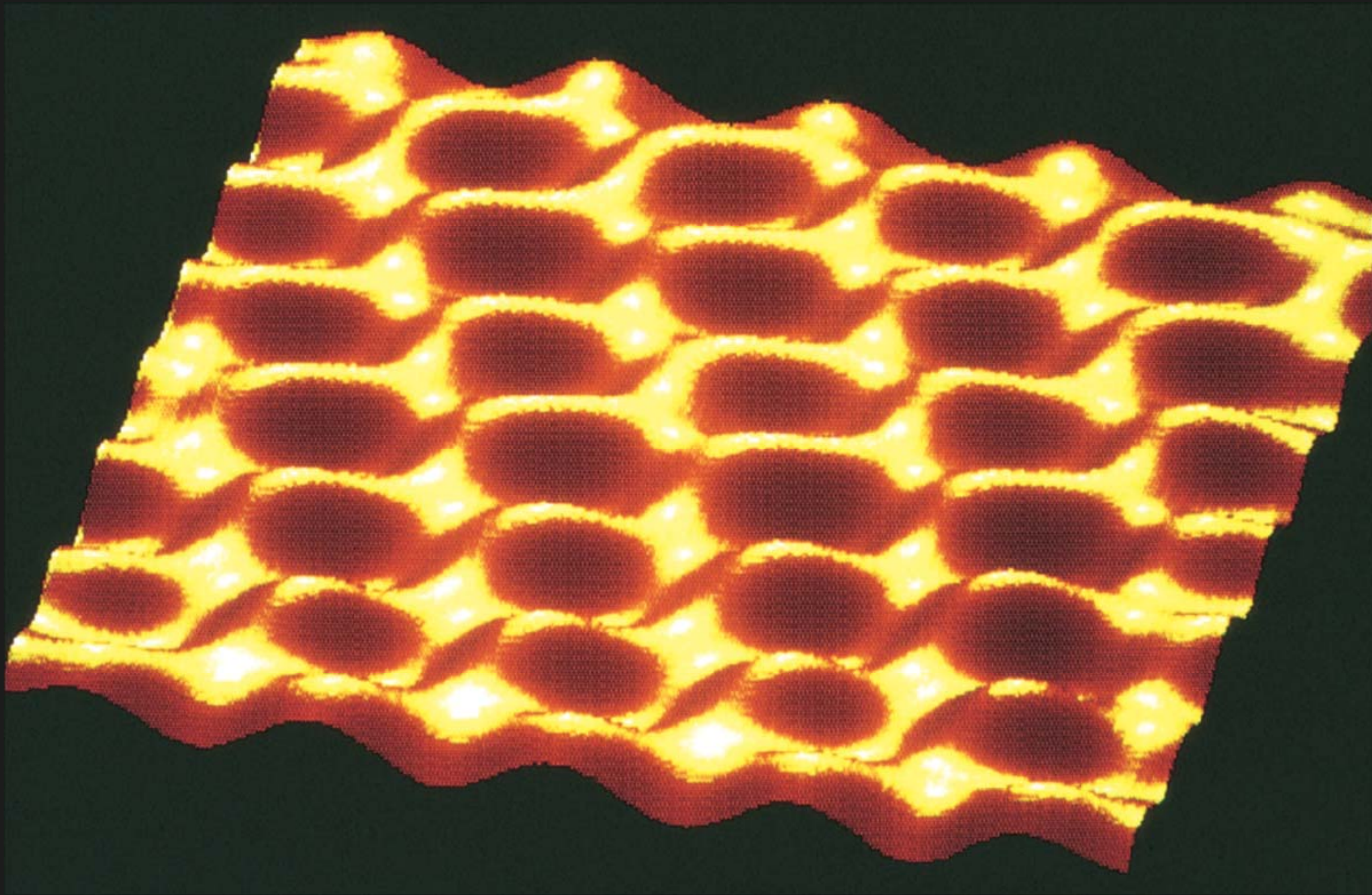


$$h\nu = \Phi + \frac{1}{2} m_{e0} v^2$$

Work function Φ

$$1 \text{ eV} = 1{,}60217733 \cdot 10^{-19} \text{ J}$$

Electrons and matter waves

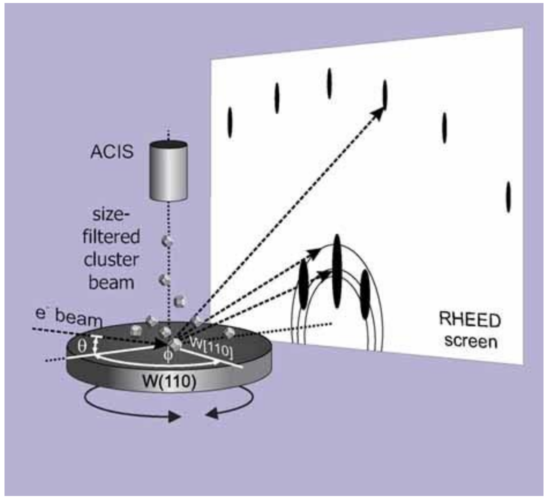


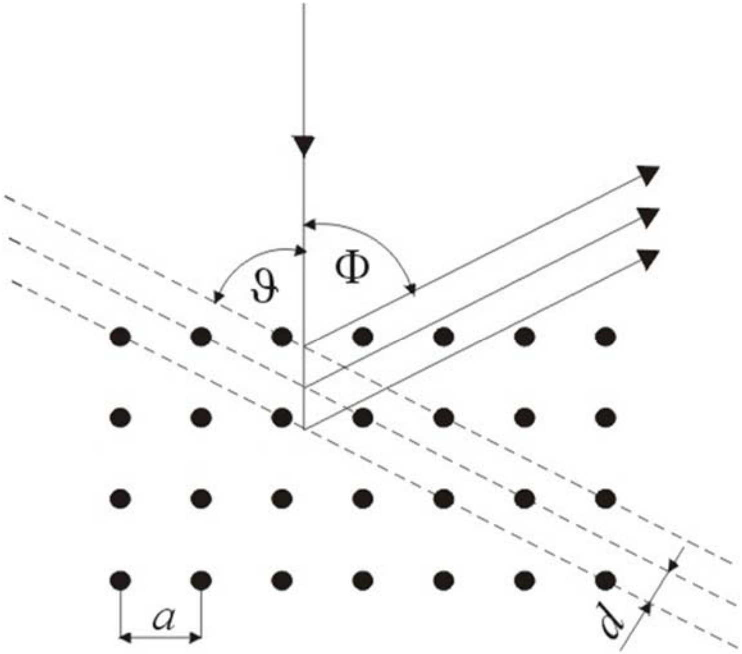
Wave of the particle

Electromagnetic waves exhibit duality:

every particle or quantum entity can be described as particle or wave

de Broglie relation $$\lambda = \frac{h}{p}$$ $$\vec{p} = \hbar\vec{k}$$



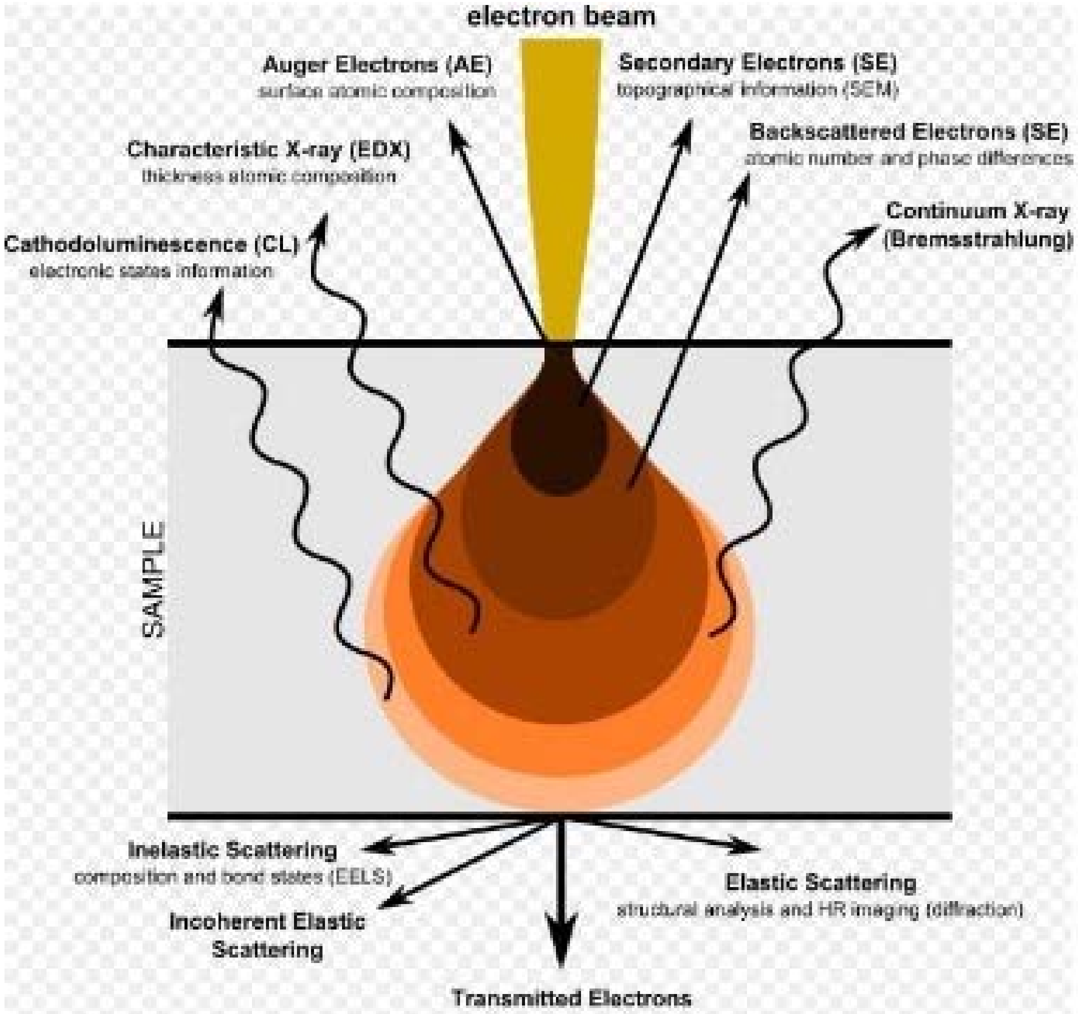


lattice parameter

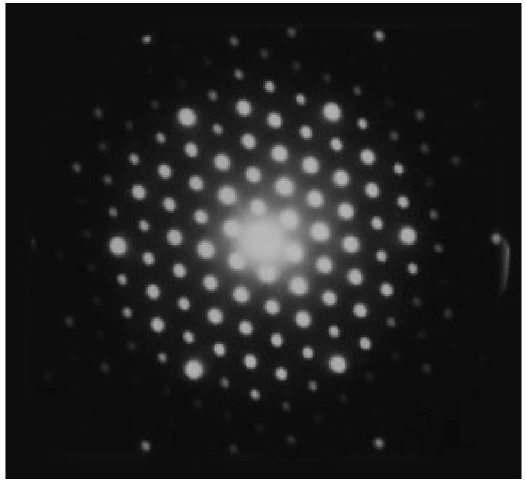
$$d = a\sin\frac{\Phi}{2}$$

$$2d\sin\vartheta = n\lambda$$

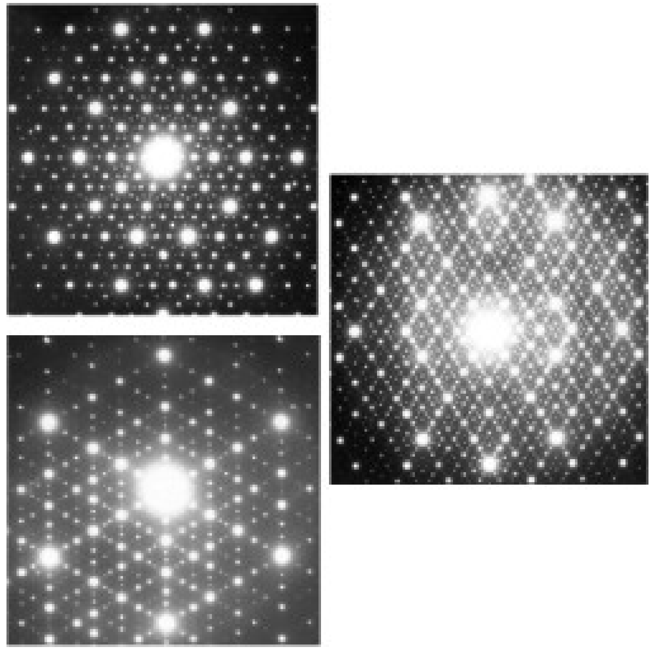
Bragg's law



https://blog.phenom-world.com/sem-electrons



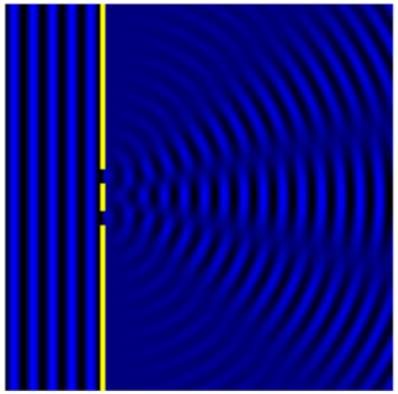
http://www.matter.org.uk/diffraction/electron/electron_diffraction.htm

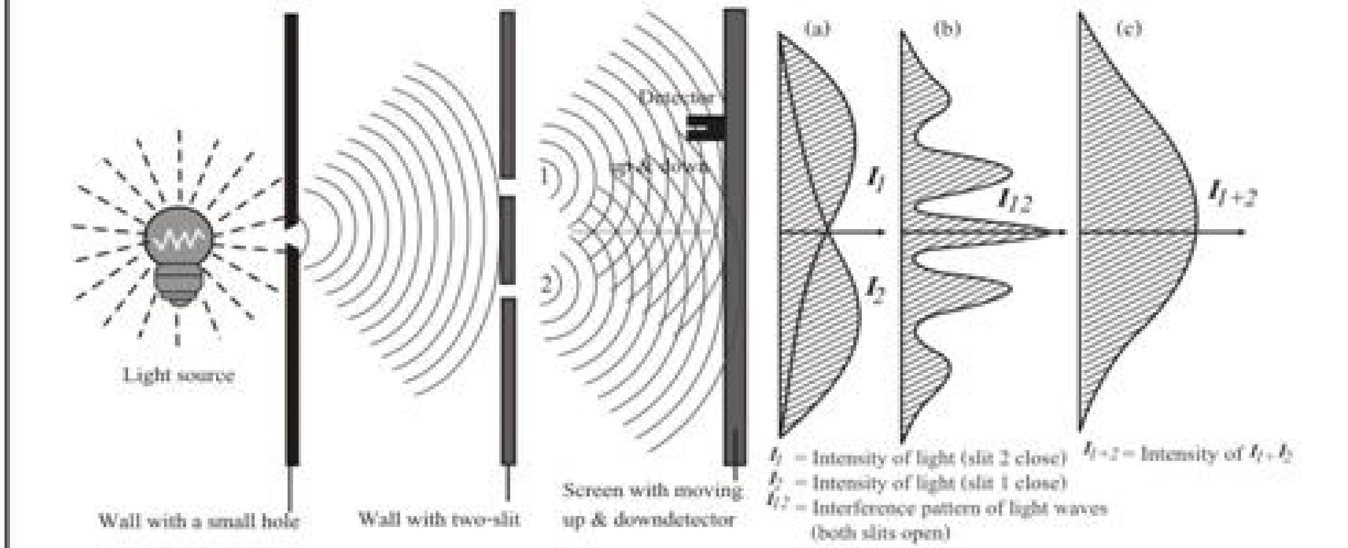


Electron diffraction patterns of the icosahedral Zn-Mg-Ho quasicrystals
http://sato.issp.u-tokyo.ac.jp/topics.html

Electron diffraction – double slit experiment

diffraction of waves

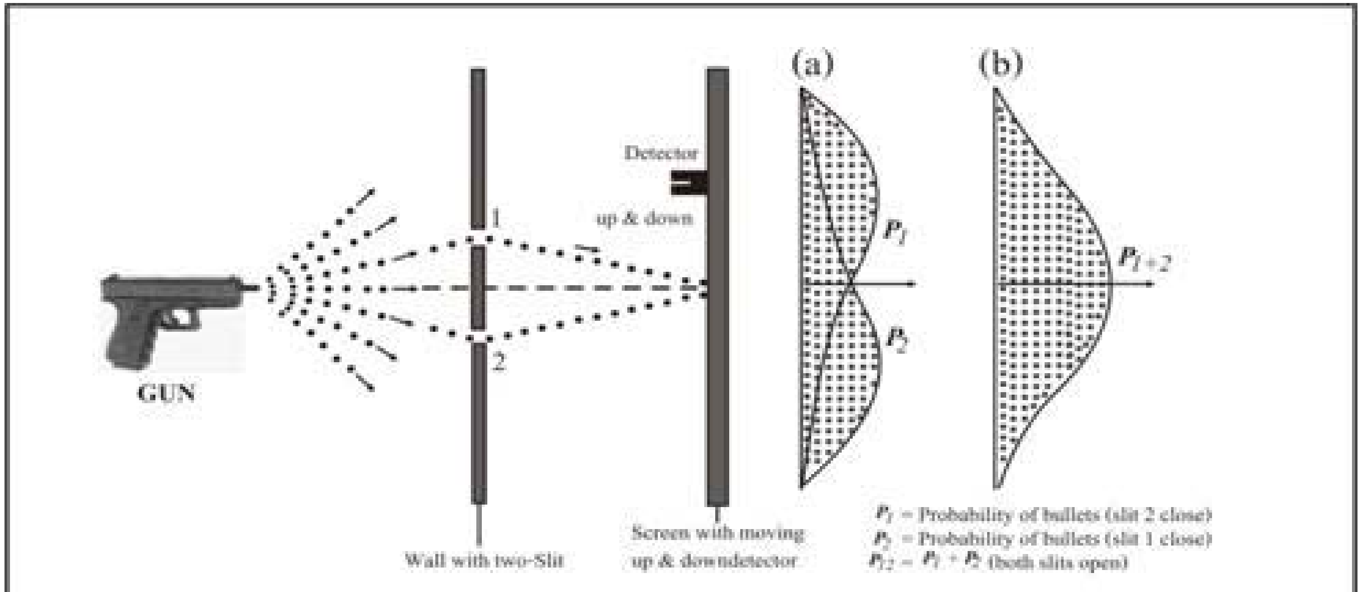




Two-slit experiment with light waves.

Electron diffraction – double slit experiment

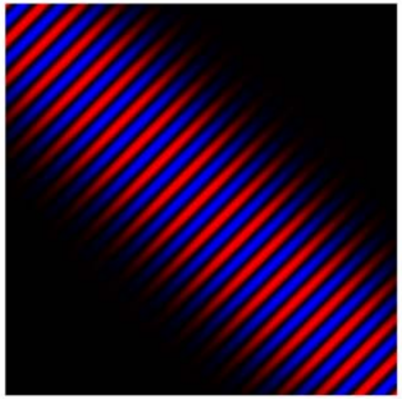
macroscopic particles

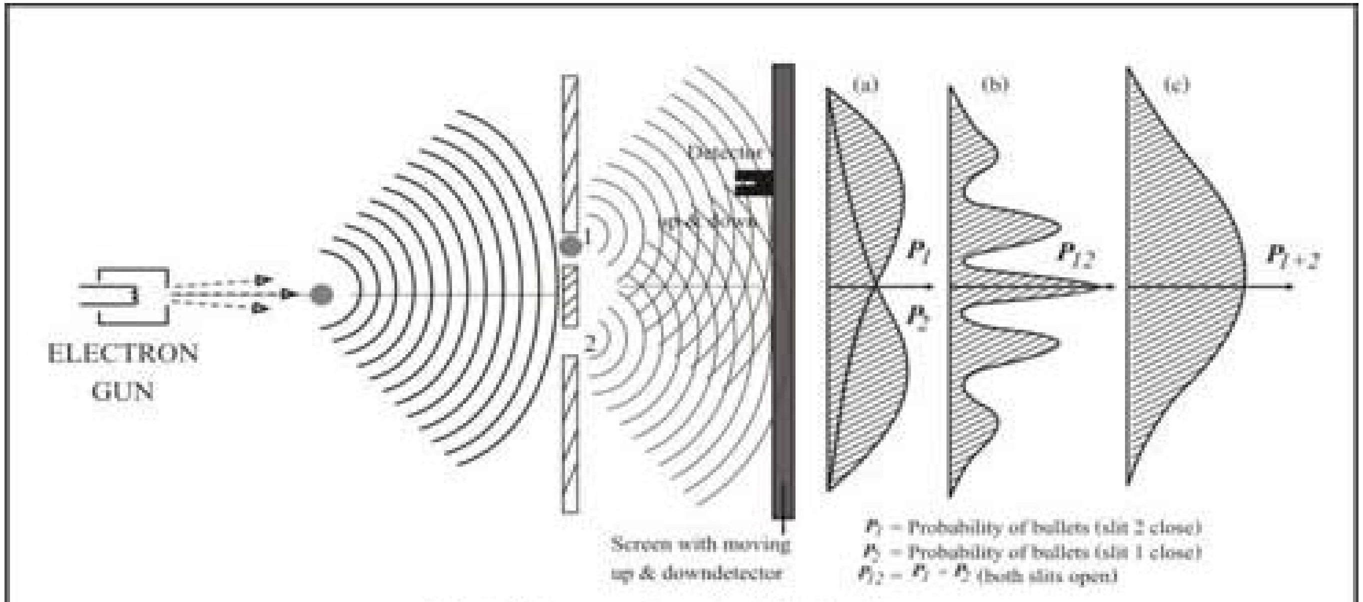


Two-slit experiment with gun's bullets.

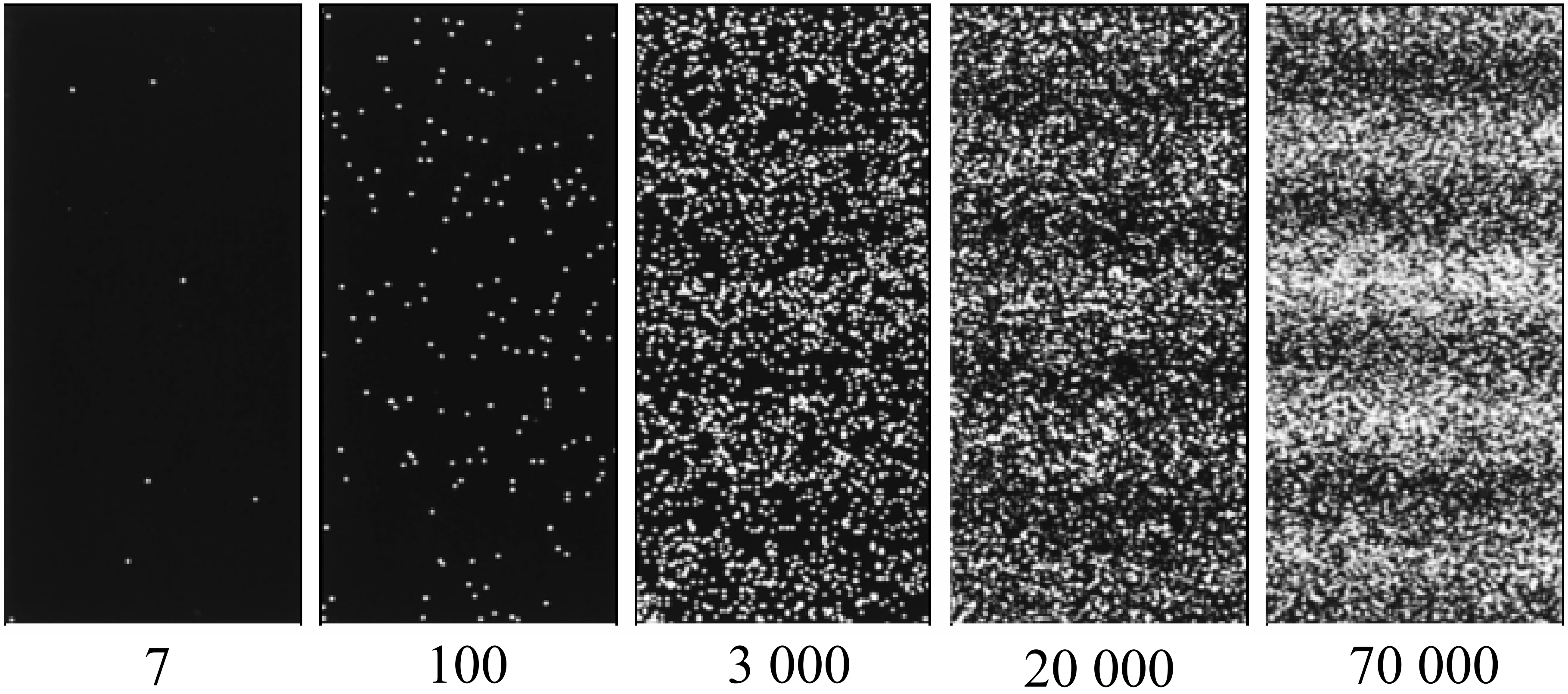
Electron diffraction – double slit experiment

diffraction of electrons



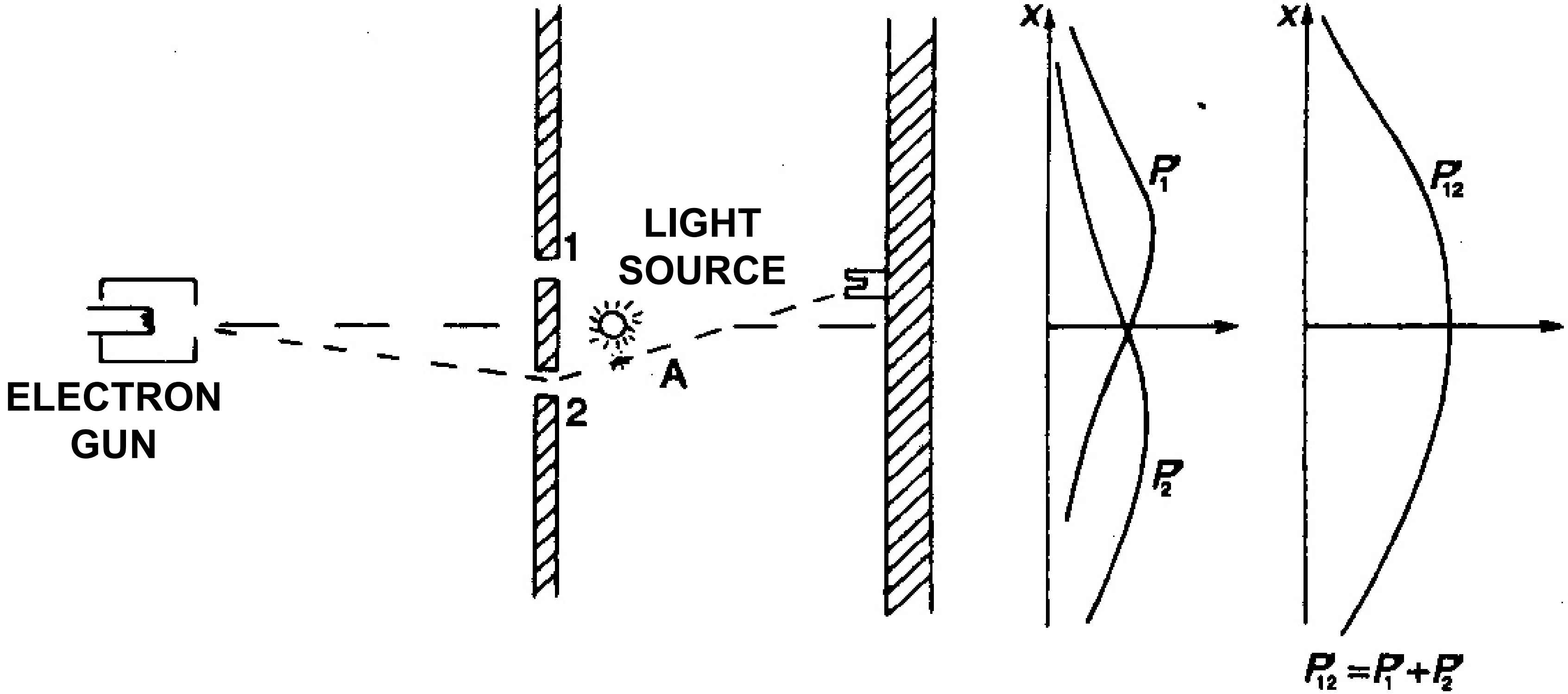


Two-Slit experiment with electrons


7
100
3 000
20 000
70 000

Complementarity principle

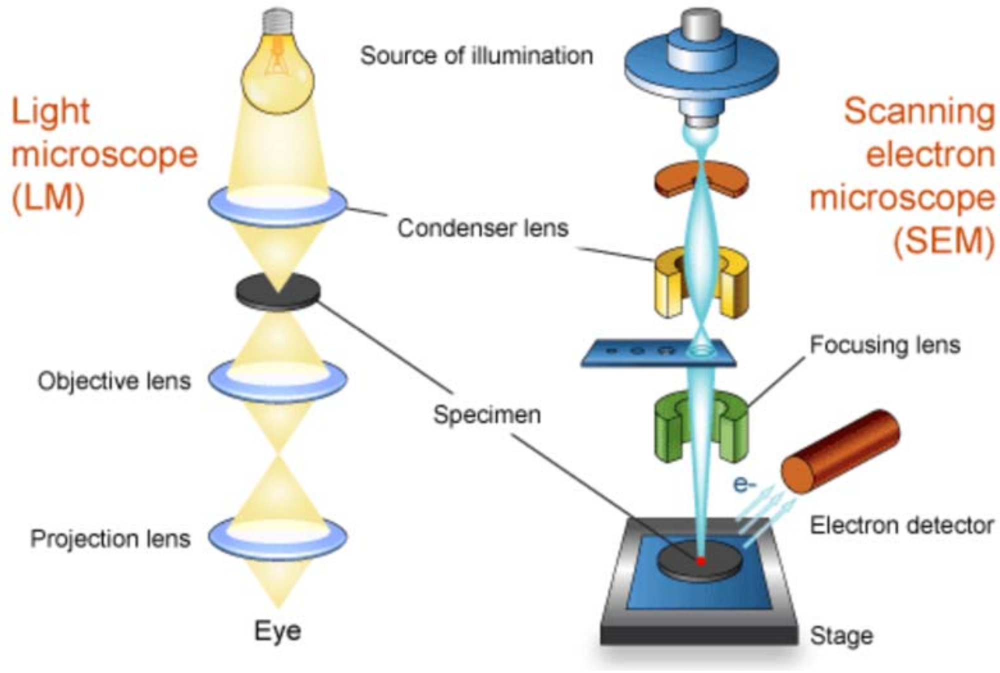
Diffraction of electrons or photon-electron scattering



no electron detection – diffraction of electrons
electron detection – loss of wave diffraction

Objects have certain pairs of complementary properties which cannot all be observed or measured simultaneously.

Electron microscopy
Electron-matter interaction
magnification $10^3 - 10^5$
resolution limit at 0.1 nm



Heisenberg's Uncertainty Principle

It is not possible to measure the position and the momentum (speed) of a particle simultaneously with unlimited precision.

$$\Delta x \, \Delta p \geq \frac{1}{2}\hbar \qquad\qquad \Delta x \, \Delta p \approx \hbar$$

Wave function and Schrödinger's equation

Wave function $$\Psi(x,y,z,t) = \psi(x,y,z)e^{-i\omega t}$$

$$E = \hbar\omega$$

$$\hbar = \frac{h}{2\pi}$$

The probability of detecting a particle in a small volume $dV = dx\,dy\,dz$ centered on a given point in a matter wave is proportional to the value of $|\Psi|^2$ at that point.

$$\Psi^2 = \Psi^*\Psi$$

$$\int_V |\Psi|^2\, dx\, dy\, dz = 1$$ probability condition

one-dimensional time-independent Schrödinger's equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + E_p(x)\psi = E\psi$$

solution – stationary states of particle

Untrapped (free) particle

$$E = E_k = \frac{p^2}{2m}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(\frac{p^2}{2m}\right)\psi = 0$$

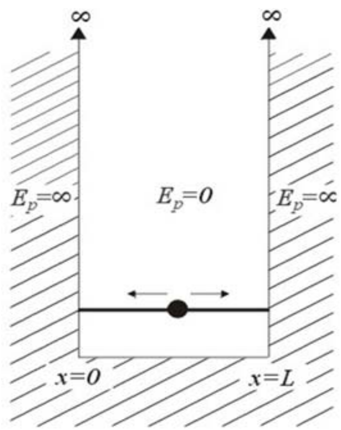
$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The probability density is a constant for any point along the x axis.

No particle position is preferred.

Energies of trapped particle

$$\frac{d^2\psi}{dx^2} + \frac{2m_e}{\hbar^2}\left[E - E_p(x)\right]\psi = 0$$



The probability density is dependent on position.

solution leads to quantization – existence of discrete states with discrete energies.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e E}}$$

$$E_k = \frac{p^2}{2m_e}$$

$$\boxed{E_n = \left(\frac{h^2}{8m_e L^2}\right)n^2}$$

$$E_n = \left(\frac{\hbar^2 \pi^2}{2m_e L^2}\right)n^2$$

quantum number

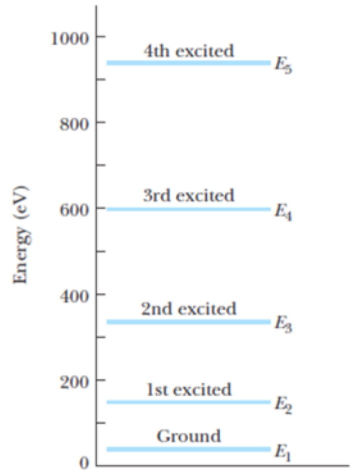
ground state n = 1

excited state

$$E_m - E_1 = h\nu$$

$$E_m - E_1 = \hbar\omega$$

energy levels



Wave functions and probability states for electron

$$\psi_n(x) = \psi_{n0}\sin\left(\frac{n\pi}{L}\right)x$$

$$\boxed{\psi_n^2(x) = \psi_{0n}^2\sin^2\left(\frac{n\pi}{L}\right)x}$$