01 – Coulomb's law (Ch. 21)

$k = 1/4\pi\varepsilon_0 = 8.99$ N m^2/C^2

1) Of the charge Q initially on a tiny sphere, a portion q is to be transferred to a second, nearby sphere. Both spheres can be treated as particles and are fixed with a certain separation. For what value of q/Q will the electrostatic force between the two spheres be maximized?

1. **THINK** After the transfer, the charges on the two spheres are $Q-q$ and q.

EXPRESS The magnitude of the electrostatic force between two charges q_1 and q_2 separated by a distance r is given by the Coulomb's law (see Eq. 21-1):

$$F = k\frac{q_1 q_2}{r^2},$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\mathrm{N\cdot m^2/C^2}$. In our case, $q_1 = Q-q$ and $q_2 = q$, so the magnitude of the force of either of the charges on the other is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}.$$

We want the value of q that maximizes the function $f(q) = q(Q-q)$.

ANALYZE Setting the derivative df/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$. Thus, $q/Q = 0.500$.

LEARN The force between the two spheres is a maximum when charges are distributed evenly between them.

3) What must be the distance between point charge $q_1 = 26.0$ μC and point charge $q_2 = -47.0$ μC for the electrostatic force between them to have a magnitude of 5.70 N?

3. **THINK** The magnitude of the electrostatic force between two charges q_1 and q_2 separated by a distance r is given by Coulomb's law.

EXPRESS Equation 21-1 gives Coulomb's law, $F = k\frac{|q_1||q_2|}{r^2}$, which can be used to solve for the distance:

$$r=\sqrt{\frac{k|q_1||q_2|}{F}}.$$

ANALYZE With $F=5.70\text{ N}$, $q_1=2.60\times10^{-6}\text{ C}$ and $q_2=-47.0\times10^{-6}\text{ C}$, the distance between the two charges is

$$r=\sqrt{\frac{k|q_1||q_2|}{F}}=\sqrt{\frac{\left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(26.0\times10^{-6}\text{C}\right)\left(47.0\times10^{-6}\text{C}\right)}{5.70\text{N}}}=1.39\text{ m}.$$

LEARN The electrostatic force between two charges falls as $1/r^2$. The same inverse-square nature is also seen in the gravitational force between two masses.

6) Two equally charged particles are held $3.2\cdot10^{-3}$ m apart and then released from rest. The initial acceleration of the first particle is observed to be 7.0 m/s^2 and that of the second to be 9.0 m/s^2. If the mass of the first particle is $6.3\cdot10^{-7}$ kg, what are (a) the mass of the second particle and (b) the magnitude of the charge of each particle?

6. (a) With a understood to mean the magnitude of acceleration, Newton's second and third laws lead to

$$m_2a_2=m_1a_1\Rightarrow m_2=\frac{\left(6.3\times10^{-7}\text{kg}\right)\left(7.0\,\text{m/s}^2\right)}{9.0\,\text{m/s}^2}=4.9\times10^{-7}\text{kg}.$$

(b) The magnitude of the (only) force on particle 1 is

$$F=m_1a_1=k\frac{|q_1||q_2|}{r^2}=\left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{|q|^2}{(0.0032\text{ m})^2}.$$

Inserting the values for m_1 and a_1 (see part (a)) we obtain $|q| = 7.1\times10^{-11}$ C.

14) Three particles are fixed on an *x* axis. Particle 1 of charge q_1 is at x = -a, and particle 2 of charge q_2 is at x = +a. If their net electrostatic force on particle 3 of charge +Q is to be zero, what must be the ratio q_1/q_2 when particle 3 is at (a) x = +0.500a and (b) x = +1.50a?

14. (a) The individual force magnitudes (acting on Q) are, by Eq. 21-1,

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1|Q}{(-a-a/2)^2}=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|Q}{(a-a/2)^2}$$

which leads to $|q_1| = 9.0\,|q_2|$. Since Q is located between q_1 and q_2, we conclude q_1 and q_2 are like-sign. Consequently, $q_1/q_2 = 9.0$.

(b) Now we have

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1|Q}{(-a-3a/2)^2}=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|Q}{(a-3a/2)^2}$$

which yields $|q_1| = 25\,|q_2|$. Now, Q is not located between q_1 and q_2; one of them must push and the other must pull. Thus, they are unlike-sign, so $q_1/q_2 = -25$.