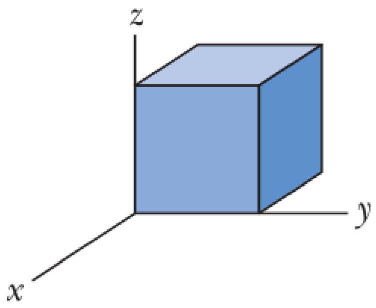
03 – Gauss Law (Ch. 23)

$k = 1/4\pi\varepsilon_0 = 8.99 \cdot 10^9$ N m^2/C^2 ; $e = 1.60 \cdot 10^{-19}$; $\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m

10) The figure shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m. It lies in a region where the nonuniform electric field is given by $\mathbf{E} = (3.00x + 4.00)\hat{\imath} + 6.00\hat{\jmath} + 7.00\hat{k}$ N/C, with x in meters. What is the net charge contained by the cube?



10. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the x dependent term only. In Si units, we have

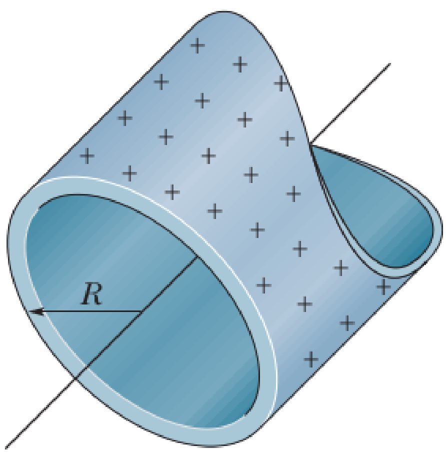
$$E_{\text{nonconstant}} = 3x\,\hat{\text{i}} \; .$$

The face of the cube located at $x = 0$ (in the yz plane) has area $A = 4$ m^2 (and it "faces" the $+\hat{\text{i}}$ direction) and has a "contribution" to the flux equal to $E_{\text{nonconstant}} A = (3)(0)(4) = 0$. The face of the cube located at $x = -2$ m has the same area A (and this one "faces" the $-\hat{\text{i}}$ direction) and a contribution to the flux:

$$-E_{\text{nonconstant}} A = -(3)(-2)(4) = 24\ \text{N}\cdot\text{m/C}^2.$$

Thus, the net flux is $\Phi = 0 + 24 = 24$ N·m/C^2. According to Gauss' law, we therefore have $q_{\text{enc}} = \varepsilon_0 \Phi = 2.13 \times 10^{-10}$ C.

24) The figure shows a section of a long, thin-walled metal tube of radius R = 3.00 cm, with a charge per unit length of $\lambda = 2.00\ 10^{-8}$ C/m. What is the magnitude E of the electric field at radial distance (a) r = R/2.00 and (b) r = 2.00R? (c) Graph E versus r for the range r = 0 to 2.00R.

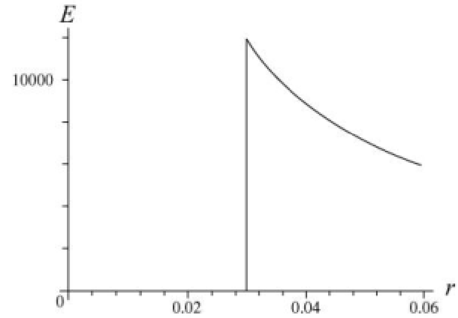


24. We imagine a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry $\oint_A \vec{E}\cdot d\vec{A} = 2\pi r E = \frac{q_{\text{enc}}}{\varepsilon_0}$.

(a) For $r < R$, $q_{\text{enc}} = 0$, so $E = 0$.

(b) For $r > R$, $q_{\text{enc}} = \lambda$, so $E(r) = \lambda / 2\pi r\varepsilon_0$. With $\lambda = 2.00\times10^{-8}$ C/m and $r = 2.00R = 0.0600$ m, we obtain

$$E = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.0600\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 5.99\times10^3\ \text{N/C}.$$



(c) The plot of E vs. r is shown to the right. Here, the maximum value is

$$E_{\max} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.030\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.2\times10^4\ \text{N/C}.$$