04 – Conductors, capacitance, energy stored in E-field (Ch. 25)

$k = 1/4\pi\varepsilon_0 = 8.99 \cdot 10^9$ N m^2/C^2 ; $e = 1.60 \cdot 10^{-19}$ C; $\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m

3) A parallel-plate capacitor has circular plates of 8.20 cm radius and 1.30 mm separation. (a) Calculate the capacitance. (b) Find the charge for a potential difference of 120 V.

3. **THINK** The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where A is the area of each plate and d is the plate separation.

EXPRESS Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates.

ANALYZE (a) Substituting the values given, the capacitance is

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{(8.85\times10^{-12}\text{ F/m})\pi(8.2\times10^{-2}\text{m})^2}{1.3\times10^{-3}\text{m}} = 1.44\times10^{-10}\text{F} = 144\text{ pF}.$$

(b) Similarly, the charge on the plate when $V = 120$ V is

$$q = (1.44\times10^{-10}\text{ F})(120\text{ V}) = 1.73\times10^{-8}\text{ C} = 17.3\text{ nC}.$$

LEARN Capacitance depends only on geometric factors, namely, the plate area and plate separation.

5) What is the capacitance of a drop that results when two mercury spheres, each of radius R = 2.00 mm, merge?

5. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \quad \Rightarrow \quad R' = 2^{1/3}R.$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 R.$$

With $R = 2.00$ mm, we obtain $C = 5.04\pi(8.85\times10^{-12}\text{ F/m})(2.00\times10^{-3}\text{m}) = 2.80\times10^{-13}\text{F}$.

31) A 2.0 mF capacitor and a 4.0 mF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

31. **THINK** The total electrical energy is the sum of the energies stored in the individual capacitors.

EXPRESS The energy stored in a charged capacitor is

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2.$$

Since we have two capacitors that are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

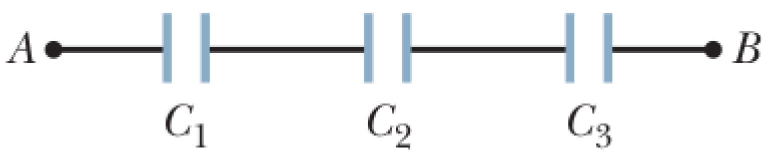
$$U_{\text{tot}} = U_1 + U_2 = \frac{1}{2}(C_1 + C_2)V^2.$$

ANALYZE Substituting the values given, we have

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\ \text{F} + 4.0\times10^{-6}\ \text{F}\right)(300\ \text{V})^2 = 0.27\ \text{J}.$$

LEARN The energy stored in a capacitor is equal to the amount of work required to charge the capacitor.

39) In the figure, C_1 = 10.0 μF C_2 = 20.0 μF and C_3 = 25.0 μF. If no capacitor can withstand a potential difference of more than 100 V without failure, what are (a) the magnitude of the maximum potential difference that can exist between points A and B and (b) the maximum energy that can be stored in the three-capacitor arrangement?



39. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 μF, then the voltage across the 20 μF capacitor is 50 V and the voltage across the 25 μF capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 25-21 or Eq. 25-22, we sum the energies on the capacitors and obtain U_{total} = 0.095 J.

$$U = \frac{q^2}{2C} \quad \text{(potential energy).} \qquad (25\text{-}21)$$

$$U = \tfrac{1}{2}CV^2 \quad \text{(potential energy).} \qquad (25\text{-}22)$$