05 – Electric current, circuits (Ch. 26-27)

$k = 1/4\pi\varepsilon_0 = 8.99 \cdot 10^9$ N m^2/C^2 ; $e = 1.60 \cdot 10^{-19}$ C; $\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m

1) During the 4.0 min a 5.0 A current is set up in a wire, how many (a) coulombs and (b) electrons pass through any cross section across the wire's width?

1. (a) The charge that passes through any cross section is the product of the current and time. Since $t = 4.0$ min $= (4.0 \text{ min})(60 \text{ s/min}) = 240$ s,

$$q = it = (5.0 \text{ A})(240 \text{ s}) = 1.2\times 10^3 \text{ C}.$$

(b) The number of electrons N is given by $q = Ne$, where e is the magnitude of the charge on an electron. Thus,

$$N = q/e = (1200 \text{ C})/(1.60 \times 10^{-19} \text{ C}) = 7.5 \times 10^{21}.$$

7) A fuse in an electric circuit is a wire that is designed to melt, and thereby open the circuit, if the current exceeds a predetermined value. Suppose that the material to be used in a fuse melts when the current density rises to 440 A/cm^2 . What diameter of cylindrical wire should be used to make a fuse that will limit the current to 0.50 A?

7. The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius (half its thickness). The magnitude of the current density vector is

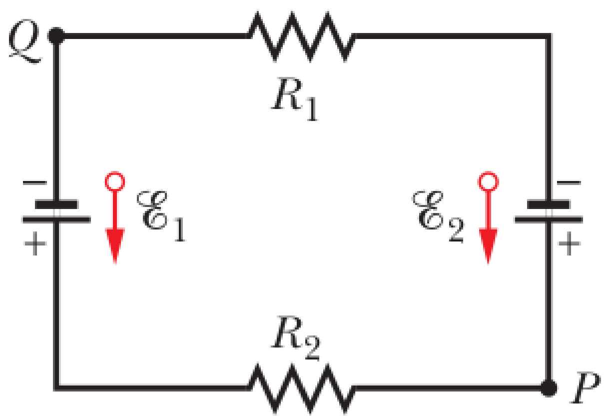
$$J = i/A = i/\pi r^2,$$

so

$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50 \text{ A}}{\pi\left(440\times 10^4 \text{ A/m}^2\right)}} = 1.9\times 10^{-4} \text{ m}.$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4} \text{ m}) = 3.8 \times 10^{-4}$ m.

2) In the figure, the ideal batteries have emfs E_1 = 150 V and E_2 = 50 V and the resistances are R_1 = 3.0 Ω and R_2 = 2.0 Ω. If the potential at P is 100 V, what is it at Q?



2. The current in the circuit is

$$i = (150\text{ V} - 50\text{ V})/(3.0\ \Omega + 2.0\ \Omega) = 20\text{ A}.$$

So from $V_Q + 150\text{ V} - (2.0\ \Omega)i = V_P$, we get

$$V_Q = 100\text{ V} + (2.0\ \Omega)(20\text{ A}) - 150\text{ V} = -10\text{ V}.$$

$$P = i(\mathscr{E} - ir) = i\mathscr{E} - i^2 r. \qquad (27\text{-}15)$$

$$P_r = i^2 r \quad \text{(internal dissipation rate).} \qquad (27\text{-}16)$$

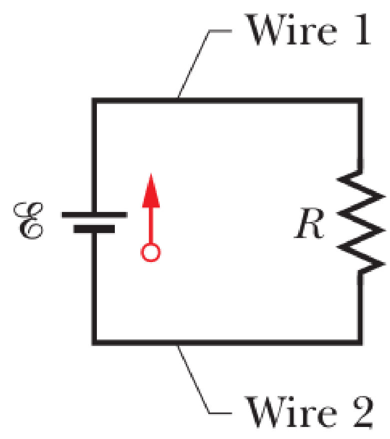
$$P_{\text{emf}} = i\mathscr{E} \quad \text{(power of emf device).} \qquad (27\text{-}17)$$

8) A certain car battery with a 12.0 V emf has an initial charge of 120 A·h. Assuming that the potential across the terminals stays constant until the battery is completely discharged, for how many hours can it deliver energy at the rate of 100 W?

8. If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and ε is the emf of the battery, then $\Delta E = q\varepsilon$. Equating the two expressions for ΔE and solving for Δt, we obtain

$$\Delta t = \frac{q\varepsilon}{P} = \frac{(120\text{ A}\cdot\text{h})(12.0\text{ V})}{100\text{ W}} = 14.4\text{ h}.$$

12) The figure shows a resistor of resistance R = 6.00 Ω connected to an ideal battery of emf E = 12.0 V by means of two copper wires. Each wire has length 20.0 cm and radius 1.00 mm. In dealing with such circuits, we generally neglect the potential differences along the wires and the transfer of energy to thermal energy in them. Check the validity of this neglect for the circuit of the figure: What is the potential difference across (a) the resistor and (b) each of the two sections of wire? At what rate is energy lost to thermal energy in (c) the resistor and (d) each section of wire?



12. (a) For each wire, $R_{\text{wire}} = \rho L/A$ where $A = \pi r^2$. Consequently, we have

$$R_{\text{wire}} = (1.69 \times 10^{-8}\,\Omega\cdot\text{m})(0.200\text{ m})/\pi(0.00100\text{ m})^2 = 0.0011\ \Omega.$$

The total resistive load on the battery is therefore

$$R_{\text{tot}} = 2R_{\text{wire}} + R = 2(0.0011\ \Omega) + 6.00\ \Omega = 6.0022\ \Omega.$$

Dividing this into the battery emf gives the current

$$i = \frac{\varepsilon}{R_{\text{tot}}} = \frac{12.0\text{ V}}{6.0022\Omega} = 1.9993\text{ A}.$$

The voltage across the $R = 6.00\ \Omega$ resistor is therefore

$$V = iR = (1.9993\text{ A})(6.00\ \Omega) = 11.996\text{ V} \approx 12.0\text{ V}.$$

(b) Similarly, we find the voltage-drop across each wire to be

$$V_{\text{wire}} = iR_{\text{wire}} = (1.9993\text{ A})(0.0011\ \Omega) = 2.15\text{ mV}.$$

(c) $P = i^2R = (1.9993\text{ A})(6.00\ \Omega)^2 = 23.98\text{ W} \approx 24.0\text{ W}.$

(d) Similarly, we find the power dissipated in each wire to be 4.30 mW.