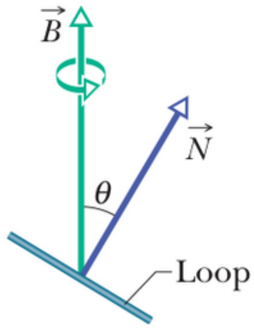
07 – Induction (Ch. 30)

$e = 1.60 \cdot 10^{-19}$ C; $m_p = 1.673 \cdot 10^{-27}$ Kg; $m_e = 9.109 \cdot 10^{-31}$ Kg



1) In the figure, a circular loop of wire 10 cm in diameter (seen edge-on) is placed with its normal **N** at an angle θ = 30° with the direction of a uniform magnetic field **B** of magnitude 0.50 T. The loop is then rotated such that **N** rotates in a cone about the field direction at the rate 100 rev/min; angle u remains unchanged during the process. What is the emf induced in the loop?

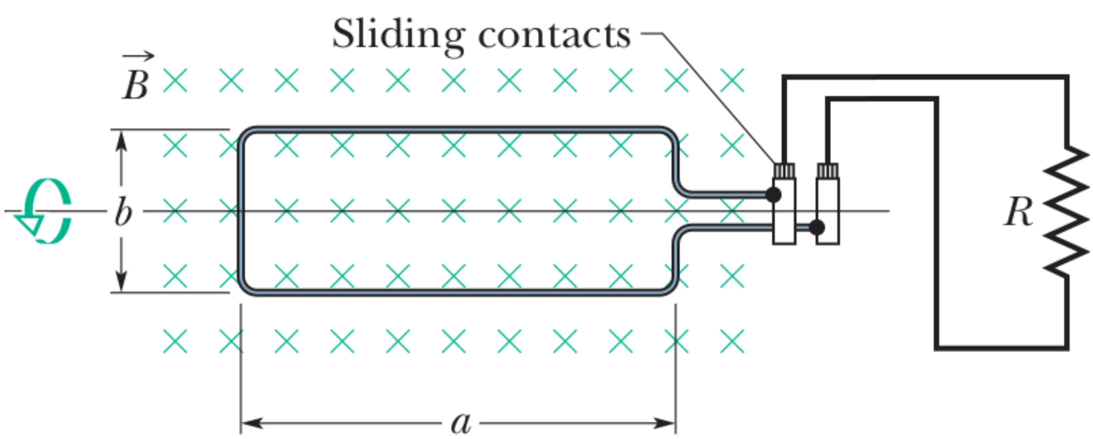
1. The flux $\Phi_B = BA\cos\theta$ does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is zero.

2) A certain elastic conducting material is stretched into a circular loop of 12.0 cm radius. It is placed with its plane perpendicular to a uniform 0.800 T magnetic field. When released, the radius of the loop starts to shrink at an instantaneous rate of 75.0 cm/s. What emf is induced in the loop at that instant?

2. Using Faraday's law, the induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(\pi r^2)}{dt} = -2\pi rB\frac{dr}{dt}$$
$$= -2\pi(0.12\text{m})(0.800\text{T})(-0.750\text{m/s})$$
$$= 0.452\text{V}.$$

11) A rectangular coil of N turns and of length *a* and width *b* is rotated at frequency *f* in a uniform magnetic field **B**, as indicated in figure. The coil is connected to co-rotating cylinders, against which metal brushes slide to make contact. (a) Show that the emf induced in the coil is given (as a function of time *t*) by $emf = 2\pi f NabB \sin(2\pi f t) = emf_0 \sin(2\pi f t)$. This is the principle of the commercial alternating-current generator. (b) What value of *Nab* gives an emf with $emf_0 = 150$ V when the loop is rotated at 60.0 rev/s in a uniform magnetic field of 0.500 T?



11. (a) It should be emphasized that the result, given in terms of $\sin(2\pi ft)$, could as easily be given in terms of $\cos(2\pi ft)$ or even $\cos(2\pi ft + \phi)$ where ϕ is a phase constant as discussed in Chapter 15. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = NBab2\pi f\sin(2\pi ft)$$

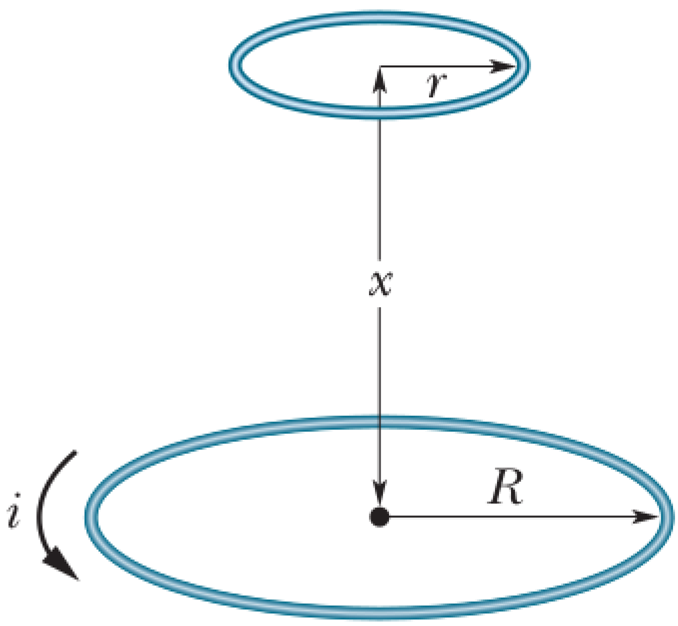
which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0 \sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0 = 2\pi f NabB$.

(b) We solve

$$\varepsilon_0 = 150\text{ V} = 2\pi f NabB$$

when $f = 60.0$ rev/s and $B = 0.500$ T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $Nab = 0.796\text{ m}^2$.

23) The figure shows two parallel loops of wire having a common axis. The smaller loop (radius r) is above the larger loop (radius R) by a distance $x >> R$. Consequently, the magnetic field due to the counterclockwise current i in the larger loop is nearly uniform throughout the smaller loop. Suppose that x is increasing at the constant rate $dx/dt = v$. (a) Find an expression for the magnetic flux through the area of the smaller loop as a function of x. In the smaller loop, find (b) an expression for the induced emf and (c) the direction of the induced current.



$$\vec{B} = \frac{\mu_0 i R^2}{2x^3}\,\hat{\mathrm{i}}$$

23. **THINK** Increasing the separation between the two loops changes the flux through the smaller loop and, therefore, induces a current in the smaller loop.

EXPRESS The magnetic flux through a surface is given by $\Phi_B = \int \vec{B}\cdot d\vec{A}$, where $\vec{B}$ is the magnetic field and $d\vec{A}$ is a vector of magnitude dA that is normal to a differential area dA. In the case where $\vec{B}$ is uniform and perpendicular to the plane of the loop, $\Phi_B = BA$.

In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Equation 29-27, with $z = x$ (taken to be much greater than R), gives $\vec{B} = \dfrac{\mu_0 i R^2}{2x^3}\hat{\mathrm{i}}$, where the $+x$ direction is upward in Fig. 30-47. The area of the smaller loop is $A = \pi r^2$.

ANALYZE (a) The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area of the smaller loop:

$$\Phi_B = BA = \frac{\pi\mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) As the smaller loop moves upward, the flux through it decreases. The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

72) Coil 1 has L_1 = 25 mH and N_1 = 100 turns. Coil 2 has L_2 = 40 mH and N 2 = 200 turns. The coils are fixed in place; their mutual inductance *M* is 3.0 mH. A 6.0 mA current in coil 1 is changing at the rate of 4.0 A/s. (a) What magnetic flux Φ_{12} links coil 1, and (b) what self-induced emf appears in that coil? (c) What magnetic flux Φ_{21} links coil 2, and (d) what mutually induced emf appears in that coil?

72. (a) The flux in coil 1 is

$$\frac{L_1 i_1}{N_1} = \frac{(25\,\text{mH})(6.0\,\text{mA})}{100} = 1.5\,\mu\text{Wb}.$$

(b) The magnitude of the self-induced emf is

$$L_1 \frac{di_1}{dt} = (25\,\text{mH})(4.0\ \text{A/s}) = 1.0\times 10^2\,\text{mV}.$$

(c) In coil 2, we find

$$\Phi_{21} = \frac{Mi_1}{N_2} = \frac{(3.0\,\text{mH})(6.0\,\text{mA})}{200} = 90\,\text{nWb}.$$

(d) The mutually induced emf is

$$\varepsilon_{21} = M\frac{di_1}{dt} = (3.0\,\text{mH})(4.0\ \text{A/s}) = 12\,\text{mV}.$$