

## 02 – Electric fields (Ch. 22)

$$k = 1/4\pi\epsilon_0 = 8.99 \cdot 10^9 \text{ N m}^2/\text{C}^2 ; e = 1.60 \cdot 10^{-19}$$

4) Two charged particles are attached to an  $x$  axis: Particle 1 of charge  $-2.00 \cdot 10^{-7} \text{ C}$  is at position  $x = 6.00 \text{ cm}$  and particle 2 of charge  $+2.00 \cdot 10^{-7} \text{ C}$  is at position  $x = 21.0 \text{ cm}$ . Midway between the particles, what is their net electric field in unit-vector notation?

4. With  $x_1 = 6.00 \text{ cm}$  and  $x_2 = 21.00 \text{ cm}$ , the point midway between the two charges is located at  $x = 13.5 \text{ cm}$ . The values of the charge are

$$q_1 = -q_2 = -2.00 \times 10^{-7} \text{ C},$$

and the magnitudes and directions of the individual fields are given by:

$$\vec{E}_1 = -\frac{|q_1|}{4\pi\epsilon_0(x-x_1)^2} \hat{i} = -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) |-2.00 \times 10^{-7} \text{ C}|}{(0.135 \text{ m} - 0.060 \text{ m})^2} \hat{i} = -(3.196 \times 10^5 \text{ N/C}) \hat{i}$$

$$\vec{E}_2 = -\frac{q_2}{4\pi\epsilon_0(x-x_2)^2} \hat{i} = -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (2.00 \times 10^{-7} \text{ C})}{(0.135 \text{ m} - 0.210 \text{ m})^2} \hat{i} = -(3.196 \times 10^5 \text{ N/C}) \hat{i}$$

Thus, the net electric field is  $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = -(6.39 \times 10^5 \text{ N/C}) \hat{i}$ .

7) Four particles form a square of edge length  $a = 5.00$  cm and have charges  $q_1 = +10.0$  nC,  $q_2 = -20.0$  nC,  $q_3 = +20.0$  nC, and  $q_4 = -10.0$  nC. In unit-vector notation, what net electric field do the particles produce at the square's center?

**7. THINK** Our system consists of four point charges that are placed at the corner of a square. The total electric field at a point is the vector sum of the electric fields of individual charges.

**EXPRESS** Applying the superposition principle, the net electric field at the center of the square is

$$\vec{E} = \sum_{i=1}^4 \vec{E}_i = \sum_{i=1}^4 \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i^2} \hat{r}_i.$$

With  $q_1 = +10$  nC,  $q_2 = -20$  nC,  $q_3 = +20$  nC, and  $q_4 = -10$  nC, the  $x$  component of the electric field at the center of the square is given by, taking the signs of the charges into consideration,

$$\begin{aligned} E_x &= \frac{1}{4\pi\epsilon_0} \left[ \frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} - \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2} \right] \cos 45^\circ \\ &= \frac{1}{4\pi\epsilon_0} \frac{1}{a^2/2} (|q_1| + |q_2| - |q_3| - |q_4|) \frac{1}{\sqrt{2}}. \end{aligned}$$

Similarly, the  $y$  component of the electric field is

$$\begin{aligned} E_y &= \frac{1}{4\pi\epsilon_0} \left[ -\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} + \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2} \right] \cos 45^\circ \\ &= \frac{1}{4\pi\epsilon_0} \frac{1}{a^2/2} (-|q_1| + |q_2| + |q_3| - |q_4|) \frac{1}{\sqrt{2}}. \end{aligned}$$

The magnitude of the net electric field is  $E = \sqrt{E_x^2 + E_y^2}$ .

**ANALYZE** Substituting the values given, we obtain

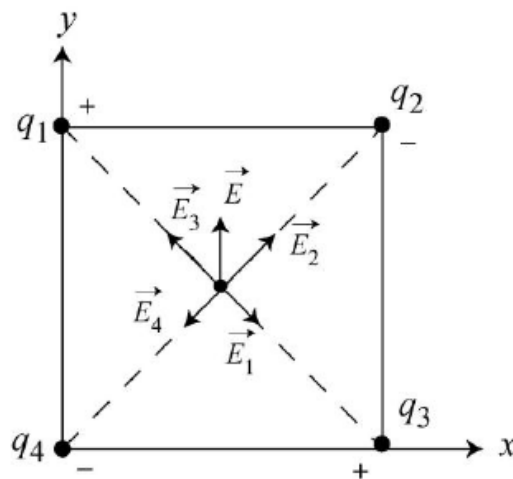
$$E_x = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{2}}{a^2} (|q_1| + |q_2| - |q_3| - |q_4|) = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{2}}{a^2} (10 \text{ nC} + 20 \text{ nC} - 20 \text{ nC} - 10 \text{ nC}) = 0$$

and

$$\begin{aligned} E_y &= \frac{1}{4\pi\epsilon_0} \frac{\sqrt{2}}{a^2} (-|q_1| + |q_2| + |q_3| - |q_4|) = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{2}}{a^2} (-10 \text{ nC} + 20 \text{ nC} + 20 \text{ nC} - 10 \text{ nC}) \\ &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(2.0 \times 10^{-8} \text{ C})\sqrt{2}}{(0.050 \text{ m})^2} \\ &= 1.02 \times 10^5 \text{ N/C}. \end{aligned}$$

Thus, the electric field at the center of the square is  $\vec{E} = E_y \hat{j} = (1.02 \times 10^5 \text{ N/C}) \hat{j}$ .

**LEARN** The net electric field at the center of the square is depicted in the figure below (not to scale). The field, pointing to the +y direction, is the vector sum of the electric fields of individual charges.



19) The figure shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole's electric field at point P, located at distance  $r \gg d$ ?

19. (a) Consider the figure below. The magnitude of the net electric field at point P is

$$|\vec{E}_{\text{net}}| = 2E_1 \sin \theta = 2 \left[ \frac{1}{4\pi\epsilon_0} \frac{q}{(d/2)^2 + r^2} \right] \frac{d/2}{\sqrt{(d/2)^2 + r^2}} = \frac{1}{4\pi\epsilon_0} \frac{qd}{[(d/2)^2 + r^2]^{3/2}}$$

For  $r \gg d$ , we write  $[(d/2)^2 + r^2]^{3/2} \approx r^3$  so the expression above reduces to

$$|\vec{E}_{\text{net}}| \approx \frac{1}{4\pi\epsilon_0} \frac{qd}{r^3}.$$

(b) From the figure, it is clear that the net electric field at point P points in the  $-\hat{j}$  direction, or  $-90^\circ$  from the +x axis.

