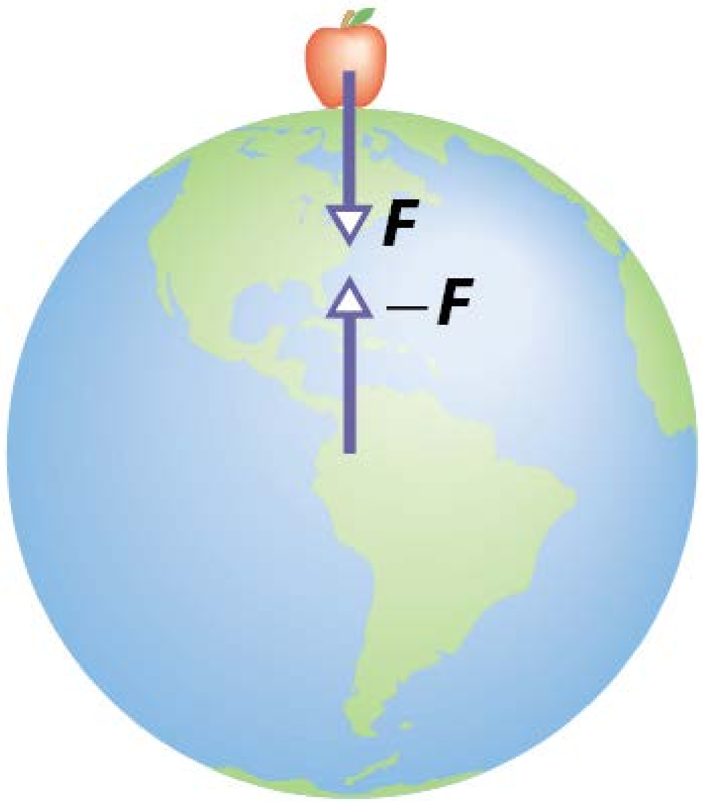
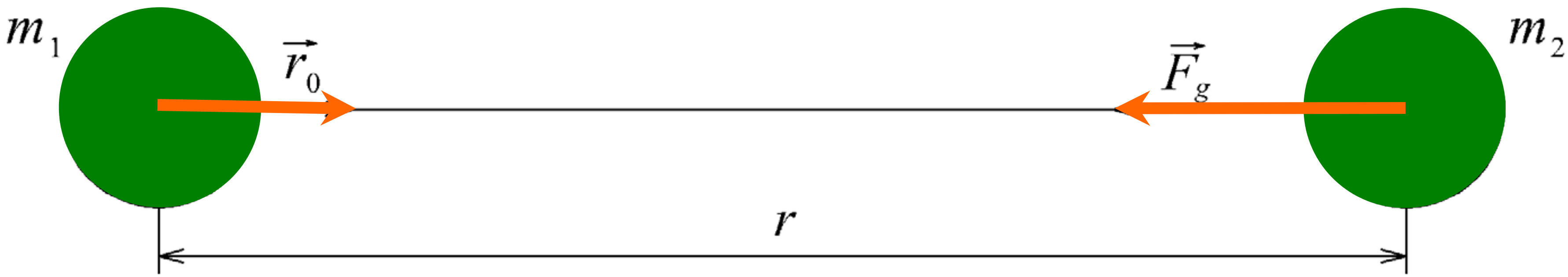
Force (vector) field



Field of Gravitation



Newton's law of gravitation

$$\vec{F}_g = -\kappa \frac{m_1 m_2}{r^2} \vec{r}_0$$

$$\kappa = 6.67 \cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}}$$

Intensity of the gravitational field

$$\vec{K} = \frac{\vec{F}_g}{m_2}$$

magnitude of a gravitational **force** caused by the mass of m_1 acting on the unit of the mass m_2

Potential energy

$$\Delta W_p = \int_{r_1}^{r_2} \vec{F}' \cdot d\vec{r} = -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

Potential of the gravitational field

$$U = \frac{W_p}{m_2}$$

A **work** of the mass m_1 needed for the displacement of the unit of the mass m_2 from the infinity to the actual position of mass m_2

$$\vec{K} = -\mathrm{grad}\, U = \left(-\frac{\partial U}{\partial x};\ -\frac{\partial U}{\partial y};\ -\frac{\partial U}{\partial z} \right)$$

$$\vec{F}_g = -\mathrm{grad}\, W_p = \left(-\frac{\partial W_p}{\partial x};\ -\frac{\partial W_p}{\partial y};\ -\frac{\partial W_p}{\partial z} \right)$$

vector operator **nabla** $\vec{\nabla} = \left(\frac{\partial}{\partial x}; \frac{\partial}{\partial y}; \frac{\partial}{\partial z}\right)$

basic operations:

$$\text{grad}\,\varphi = \vec{\nabla}\varphi = \left(\frac{\partial \varphi}{\partial x}; \frac{\partial \varphi}{\partial y}; \frac{\partial \varphi}{\partial z}\right)$$ = vector

= scalar $\varphi = \varphi(x; y; z)$

$$\text{div}\,\vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{\partial \vec{A}}{\partial x} + \frac{\partial \vec{A}}{\partial y} + \frac{\partial \vec{A}}{\partial z}$$ = scalar

= vector $\vec{A} = (A_x; A_y; A_z)$

$$\text{rot}\,\vec{A} = \vec{\nabla} \times \vec{A} = \det\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = \cdots$$ = vector

$$\vec{\nabla} \cdot \vec{\nabla} = \Delta = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$ = scalar

Laplace operator

Potential energy of the Earth gravitational field

$$\Delta W_p = -\int_{r_E}^{r_E+h} \vec{F}\cdot d\vec{r} = \kappa\, mM_E \int_{r_E}^{r_E+h} \frac{1}{r^2} dr = \frac{\kappa\, M_E mh}{r_E^2}\left(1+\frac{h}{r_E}\right)^{-1}$$

$$= mg_0 h\left(1+\frac{h}{r_E}\right)^{-1} \approx mg_0 h\left(1-\frac{h}{r_E}\right)$$

$$\frac{\kappa\, M_E}{r_E^2} = \frac{F_g}{m} = g_0$$

near the Earth's surface

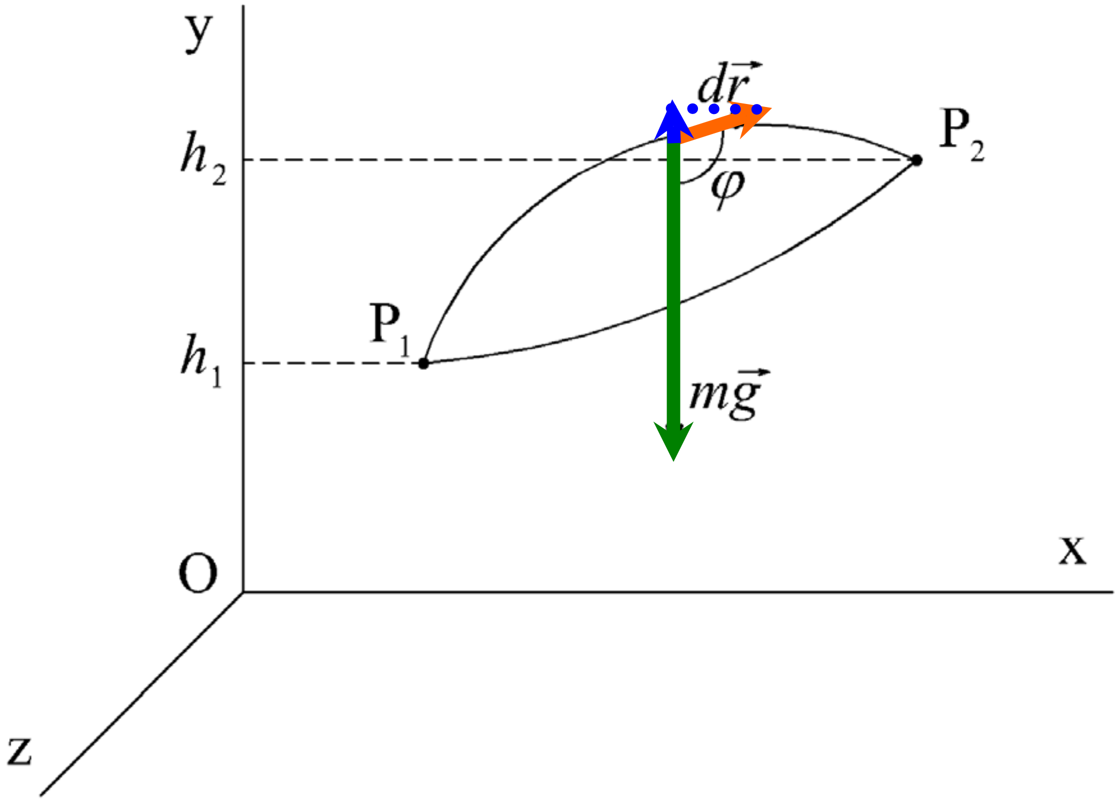
Conservative forces

stationary

$$\oint_l \vec{F}\cdot d\vec{r} = 0$$

potential

Field of Gravity



$$A = mg\int_{P_1}^{P_2} dr\cos\varphi$$

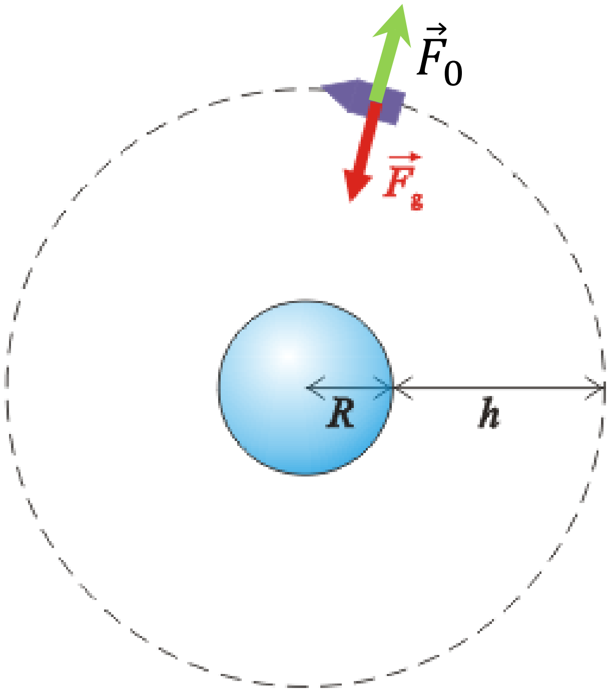
$$dr\cos(\pi - \varphi) = dh$$

$$-dr\cos\varphi = dh$$

$$A = mg(h_1 - h_2)$$

$$A = -\Delta W_p = -mg\Delta h$$

Satellites motion principle



$$F_g = F_o$$

$$\kappa \frac{M_E m}{(R_E + h)^2} = m \frac{v^2}{(R_E + h)}$$

$$v = \omega(R_E + h) = \frac{2\pi}{T}(R_E + h)$$

Orbital speed and Escape velocity