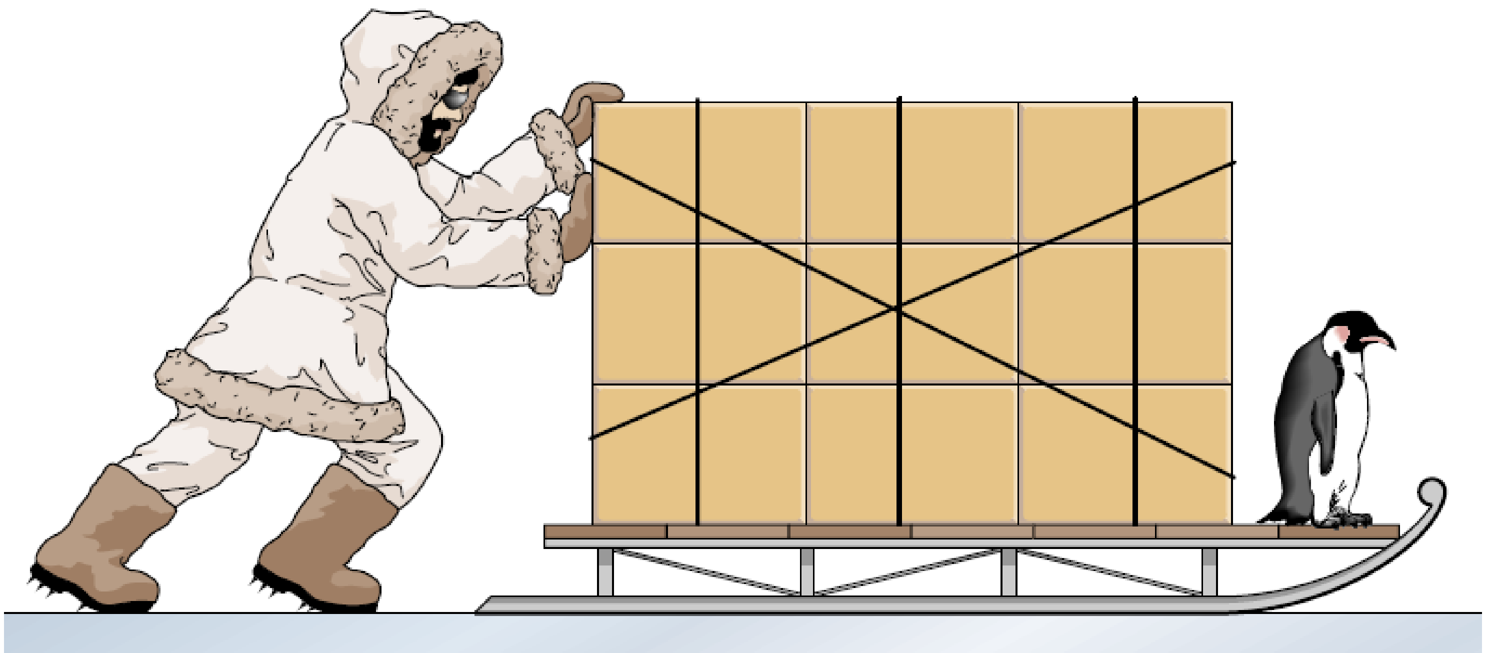
Newtonian mechanics



Mass point dynamics

Newton’s laws

1. If no force acts on a body, the body’s velocity cannot change; that is, the body cannot accelerate. (= **law of inertia**)

2. The net force on a body is equal to the product of the body’s mass and its acceleration.(= **law of force**)

3. When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

Newton’s second law

$$\vec{F} = m\vec{a}$$

$$v << c$$

$$m \neq m(t)$$

Newtonian mechanics

$$\vec{p} = m\vec{v}$$

linear momentum

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt$$

impulse due to force

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1 = m\vec{v}_2 - m\vec{v}_1$$

Newton's third law

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12} = m_1 \frac{d\vec{v}_1}{dt} \qquad \vec{F}_{21} = m_2 \frac{d\vec{v}_2}{dt}$$

$$0 = m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = \frac{d}{dt}\left(m_1\vec{v}_1 + m_2\vec{v}_2\right)$$

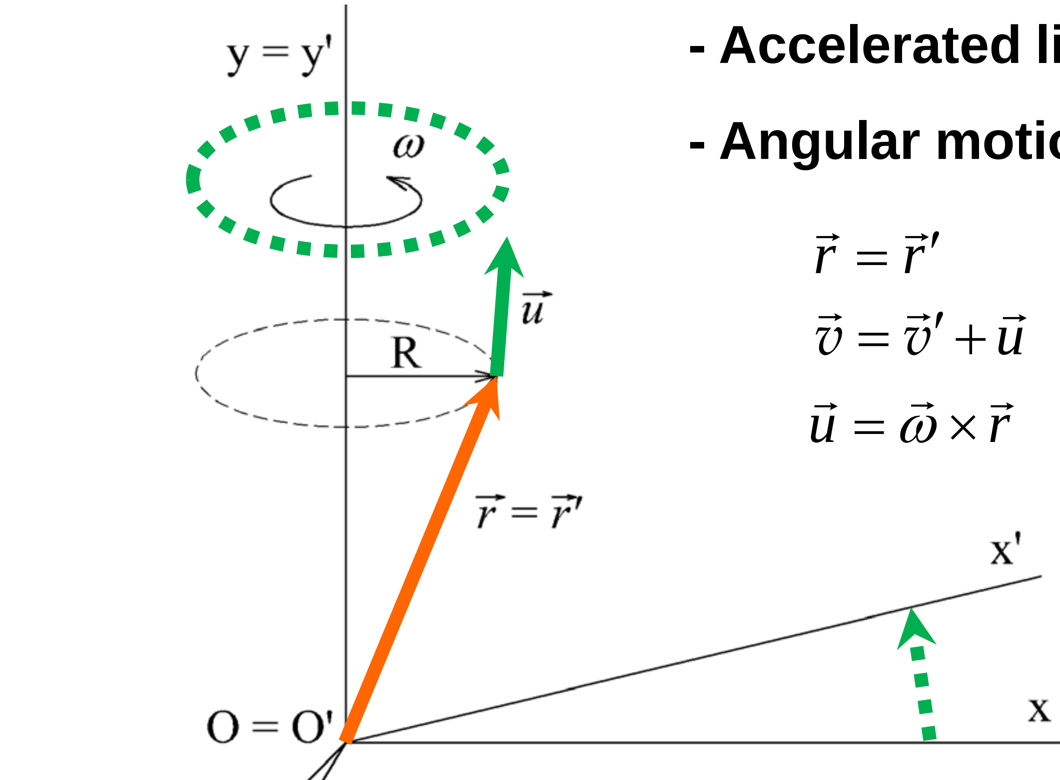
$$\boldsymbol{m_1\vec{v}_1 + m_2\vec{v}_2 = \overrightarrow{const}}$$

conservation of linear momentum

Motion equation (linear motion)

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{F} = m\frac{d^2\vec{r}}{dt^2}$$

Non-inertial reference frame



- Accelerated linear motion

- Angular motion (even uniform)

$$\vec{r} = \vec{r}'$$

$$\vec{v} = \vec{v}' + \vec{u}$$

$$\vec{u} = \vec{\omega} \times \vec{r}$$

Newton's laws are <u>not valid</u>

$$\vec{F}' = \vec{F} + \vec{F}_s$$

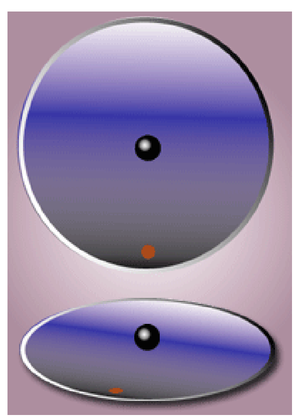
fictitious (inertial) forces

$$\vec{F}' = m\vec{a}' = m\vec{a} + \vec{F}^* + \vec{F}_C + \vec{F}_o$$

$$\vec{F}_o = -m\left[\vec{\omega} \times \left(\vec{\omega} \times \vec{r}'\right)\right]$$

$$\vec{F}_C = -2m\left(\vec{\omega} \times \vec{v}'\right)$$

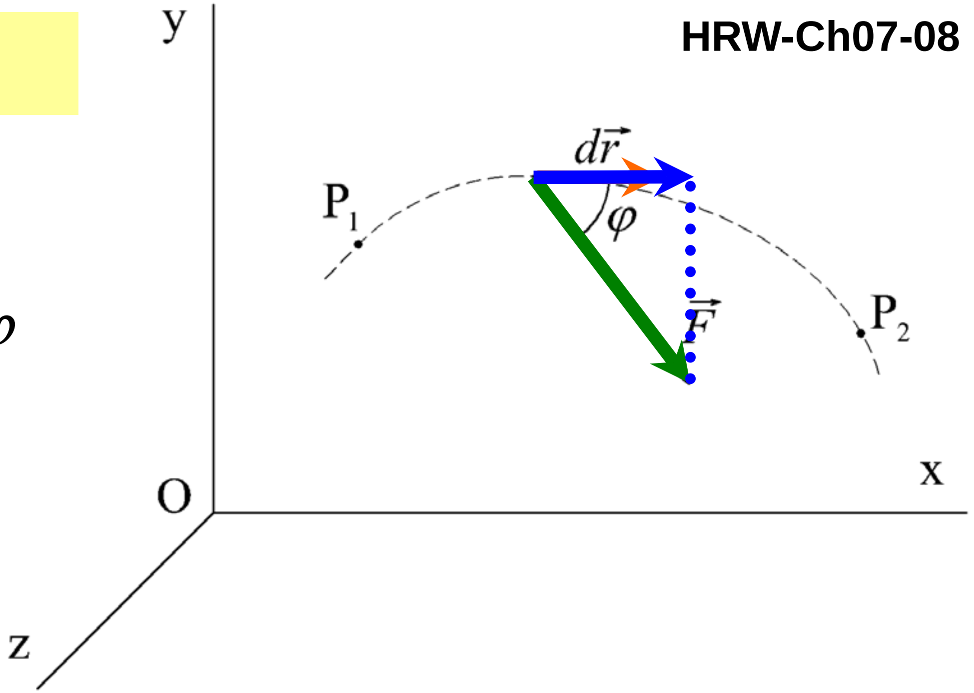
Foucault pendulum



Work and Energy

$$dA = \vec{F} \cdot d\vec{r}$$

$$A = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} = \int_{P_1}^{P_2} F\, dr \cos\varphi$$



conservative force $\oint_l \vec{F} \cdot d\vec{r} = 0$

$$A = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} = m \int_{P_1}^{P_2} \frac{d\vec{v}}{dt} \cdot d\vec{r} = m \int_{P_1}^{P_2} \left[\left(\frac{d\vec{v}}{dt} \right)_t + \left(\frac{d\vec{v}}{dt} \right)_n \right] \cdot d\vec{r} = m \int_{P_1}^{P_2} \left(\frac{d\vec{v}}{dt} \right)_t \cdot d\vec{r}$$

$$= m \int_{t_1}^{t_2} dv \frac{dr}{dt} = m \int_{v_1}^{v_2} v\, dv$$

kinetic energy

$$A = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$mgh_1 - mgh_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$