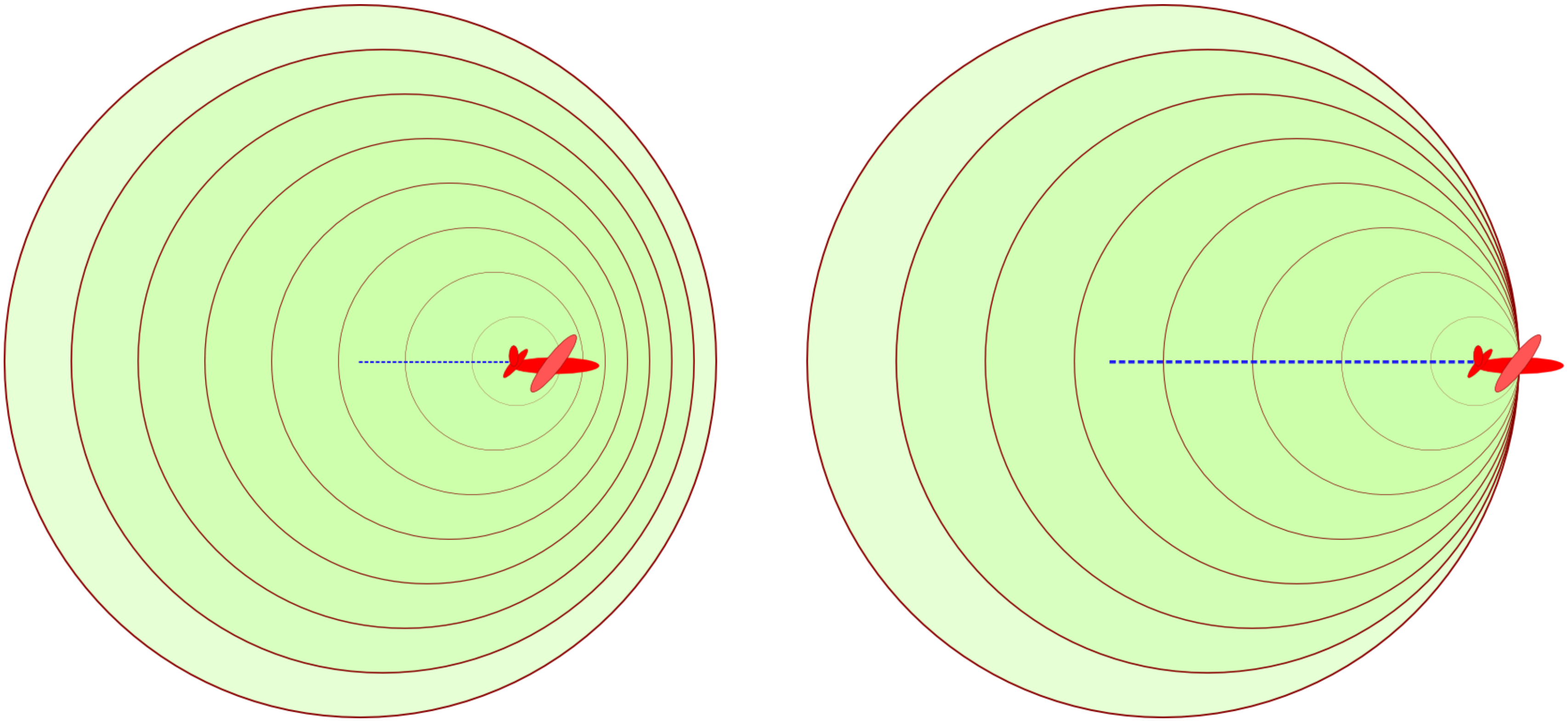
Waves

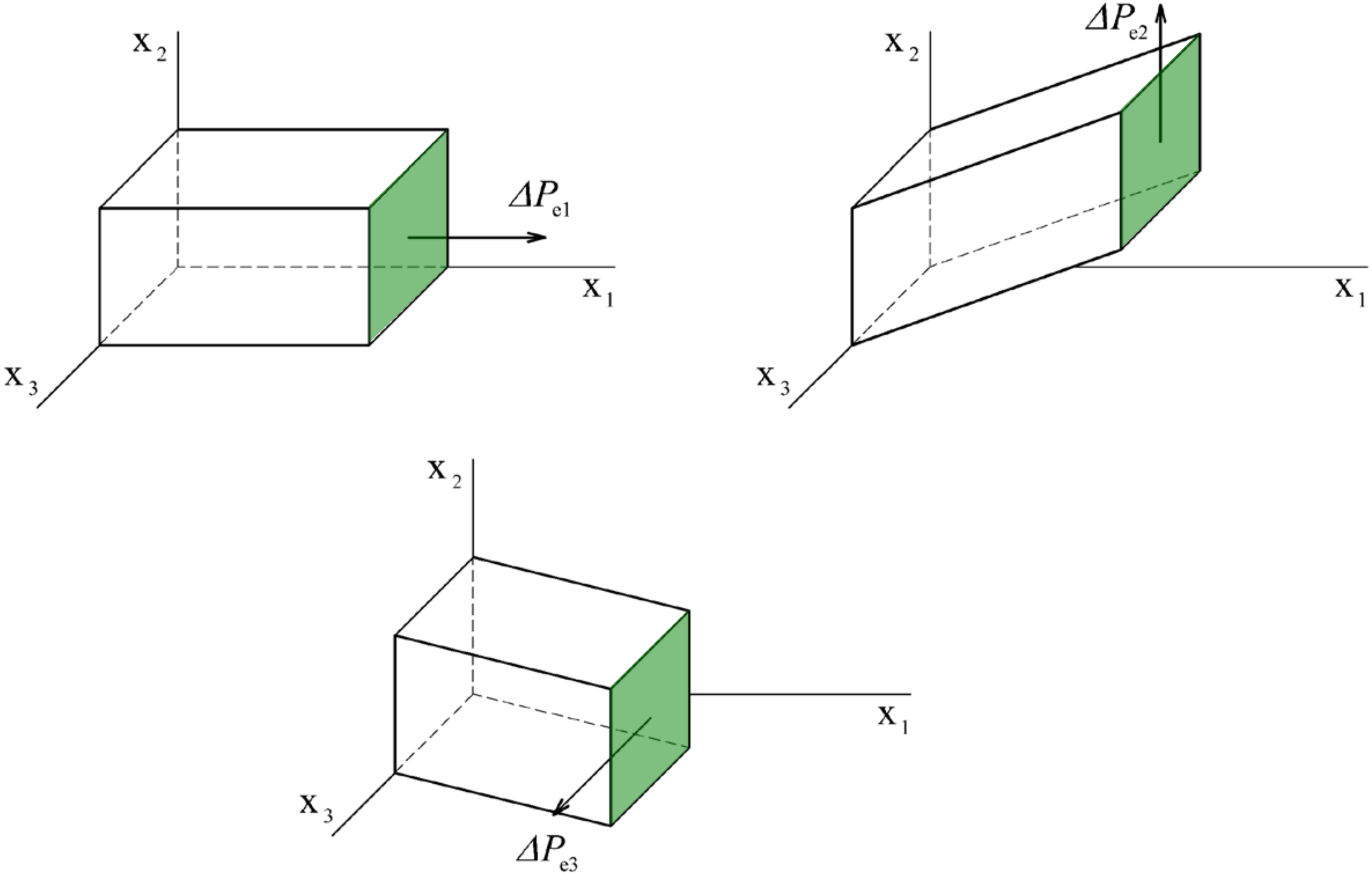


Waves

elastic wave - transversal

- longitudinal

surface force causing elastic wave generation $\Delta\vec{P}_e$



Wave Equation

$$\Delta m\,\vec{a} = \Delta \vec{P}_e$$

$$\rho\vec{a} = \vec{f}_e$$

$$\rho\frac{\partial^2 u_i}{\partial t^2} = f_{ei}$$

$$f_{ei} = \sum_{k=1}^{3}\frac{\partial \sigma_{ki}}{\partial x_k}$$

$$f_{e1} = \frac{\partial \sigma_{11}}{\partial x_1} \qquad \sigma_{11} = E\frac{\partial u_1}{\partial x_1} \qquad f_{e1} = E\frac{\partial^2 u_1}{\partial x_1^2}$$

$$f_{e2} = \frac{\partial \sigma_{12}}{\partial x_1} \qquad \sigma_{12} = G\frac{\partial u_2}{\partial x_1} \qquad f_{e2} = G\frac{\partial^2 u_2}{\partial x_1^2}$$

$$\frac{\partial^2 u_1}{\partial x_1^2} = \frac{\rho}{E}\frac{\partial^2 u_1}{\partial t^2}$$

$$\frac{\partial^2 u_2}{\partial x_1^2} = \frac{\rho}{G}\frac{\partial^2 u_2}{\partial t^2}$$

$$\frac{\partial^2 u_3}{\partial x_1^2} = \frac{\rho}{G}\frac{\partial^2 u_3}{\partial t^2}$$

$$u_i(x_i,t) = u_0\sin\omega\left(t - \frac{x_i}{c_i}\right)$$

$$u_i(x_i,t) = u_0\sin 2\pi\left(\frac{t}{T} - \frac{x_i}{c_i}\right)$$

$$u_1(x_i,t) = u_0\sin\left(\omega t - k_i x_i\right)$$

$$c_1 = \sqrt{\frac{E}{\rho}}, \qquad c_2 = c_3 = \sqrt{\frac{G}{\rho}}$$

$$T = \frac{1}{f} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

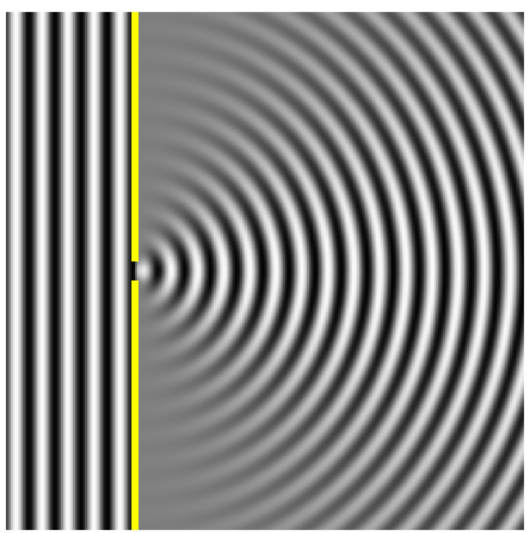
Wave Propagation

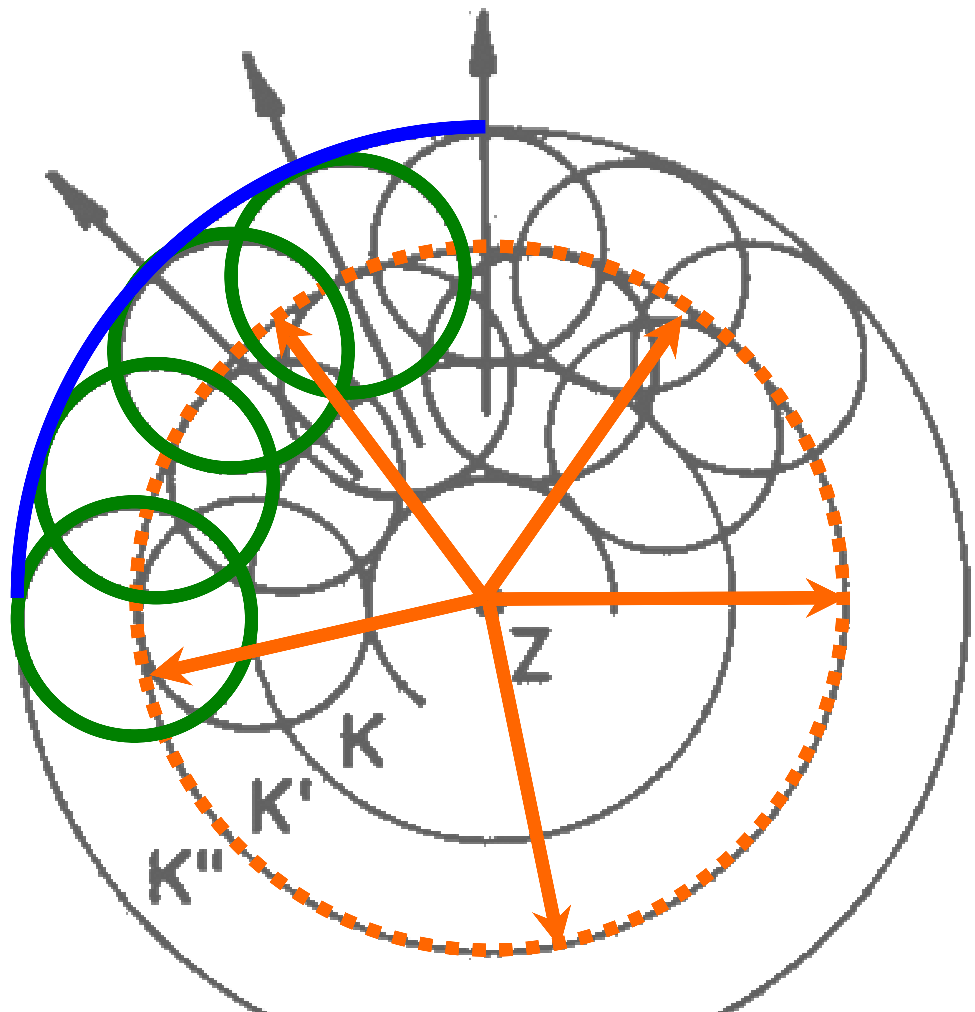
isotropic and homogeneous environment

Huygens principle

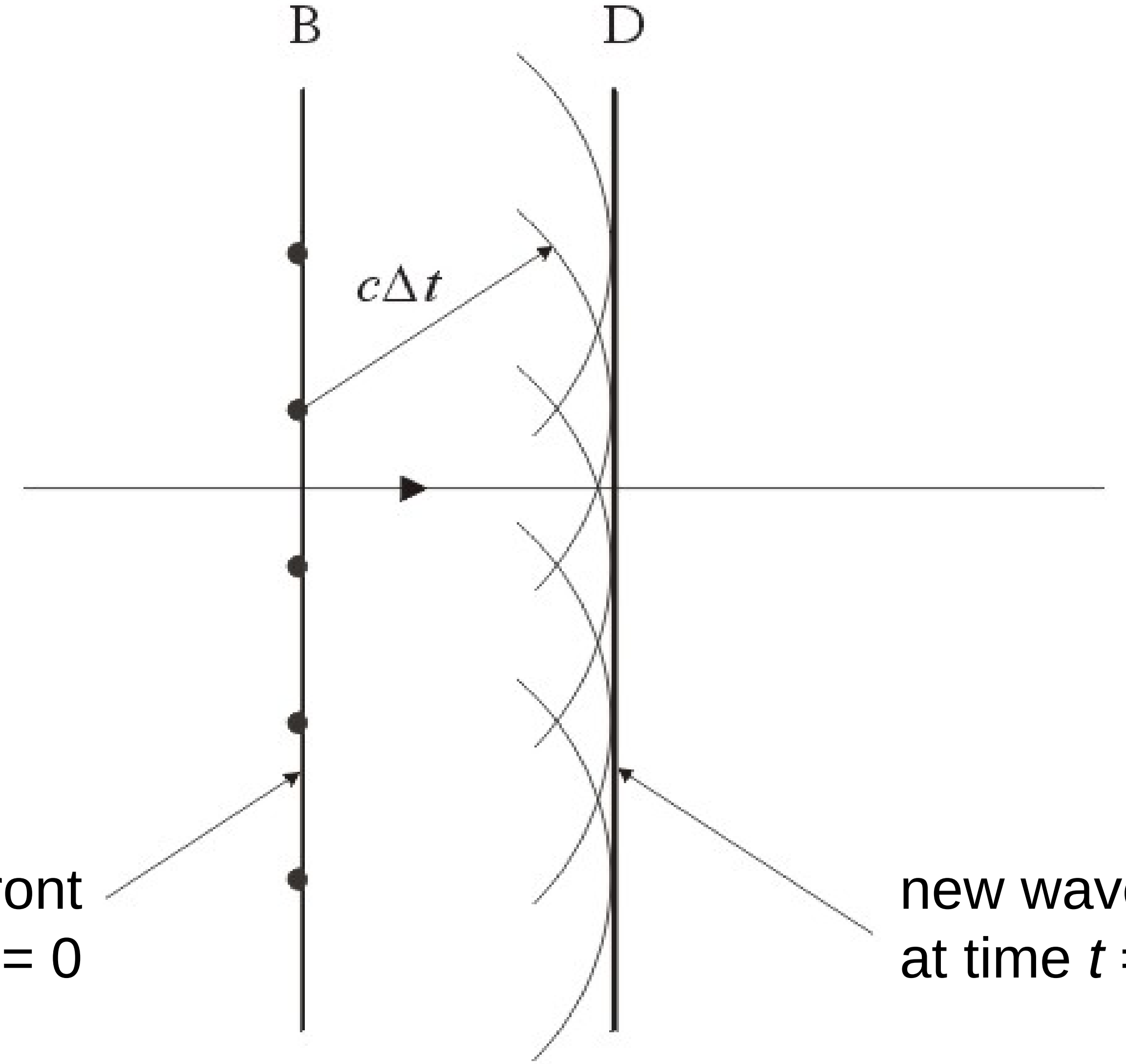
wave front

every point on a wave front is a source of new spherical waves





Plane Wave

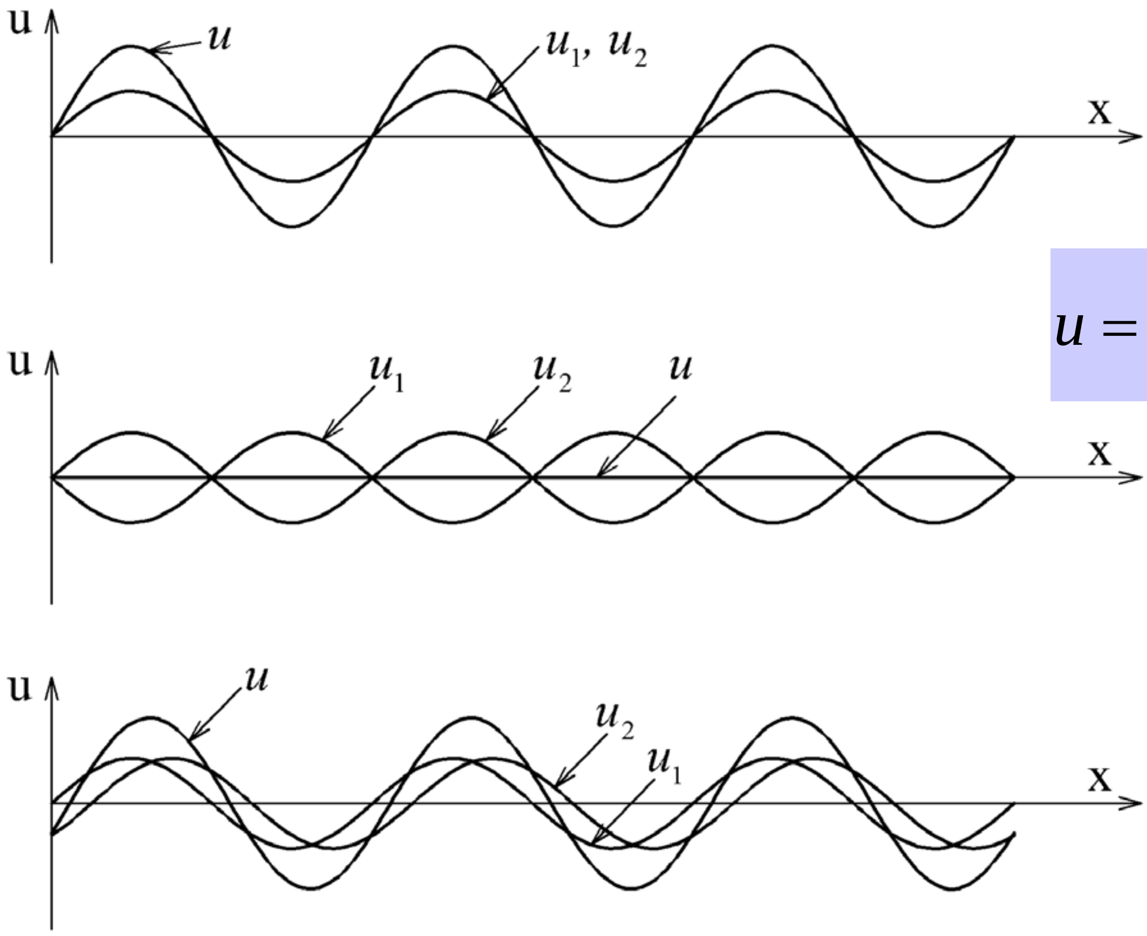


wave front
at time t = 0

new wave front
at time t = Δt

Interference

wave superposition principle

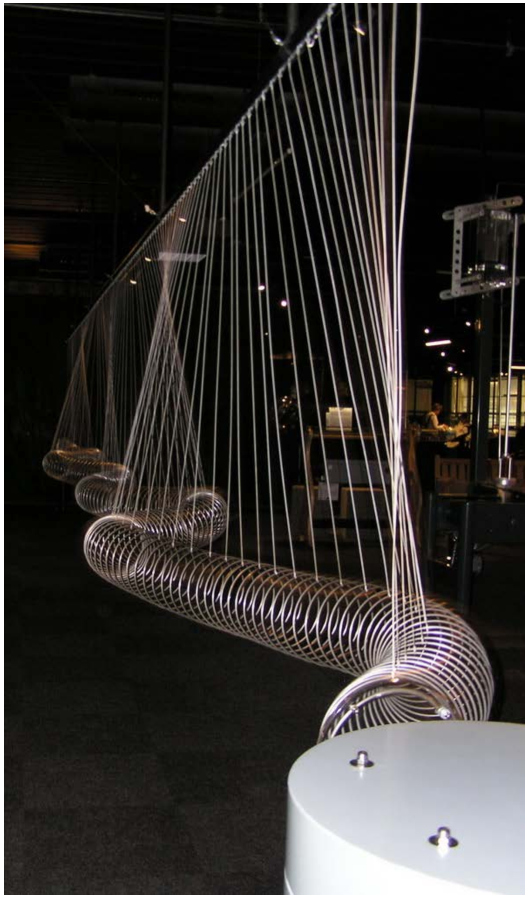


$$u_1 = u_0 \sin(\omega t - kx)$$

$$u_2 = u_0 \sin(\omega t - kx - \Phi)$$

$$u = 2u_0 \cos\frac{\Phi}{2} \sin(\omega t - kx - \frac{\Phi}{2})$$

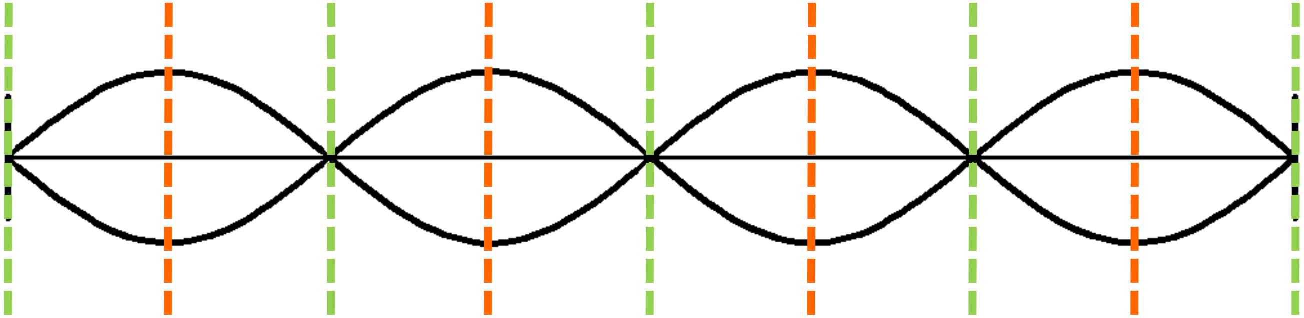
Standing waves



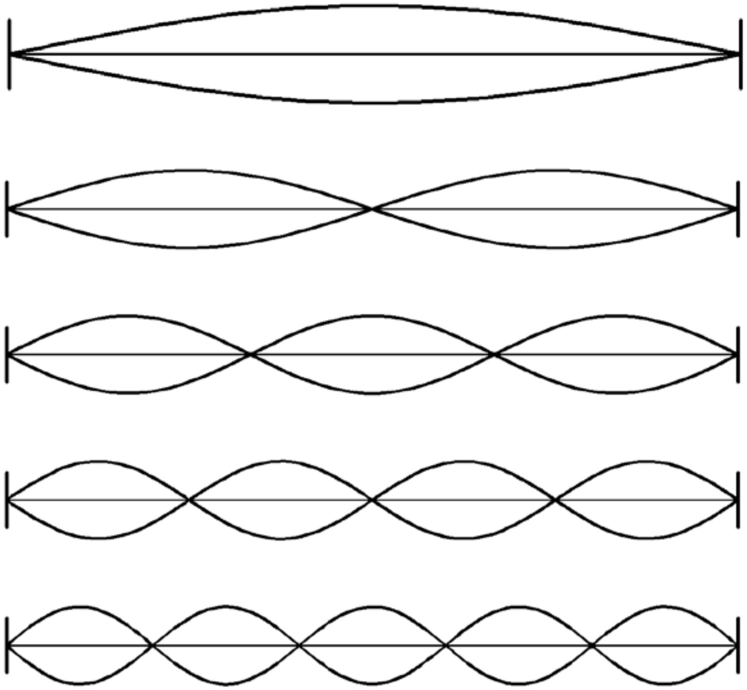
$$u_1 = u_0 \sin(\omega t - kx)$$

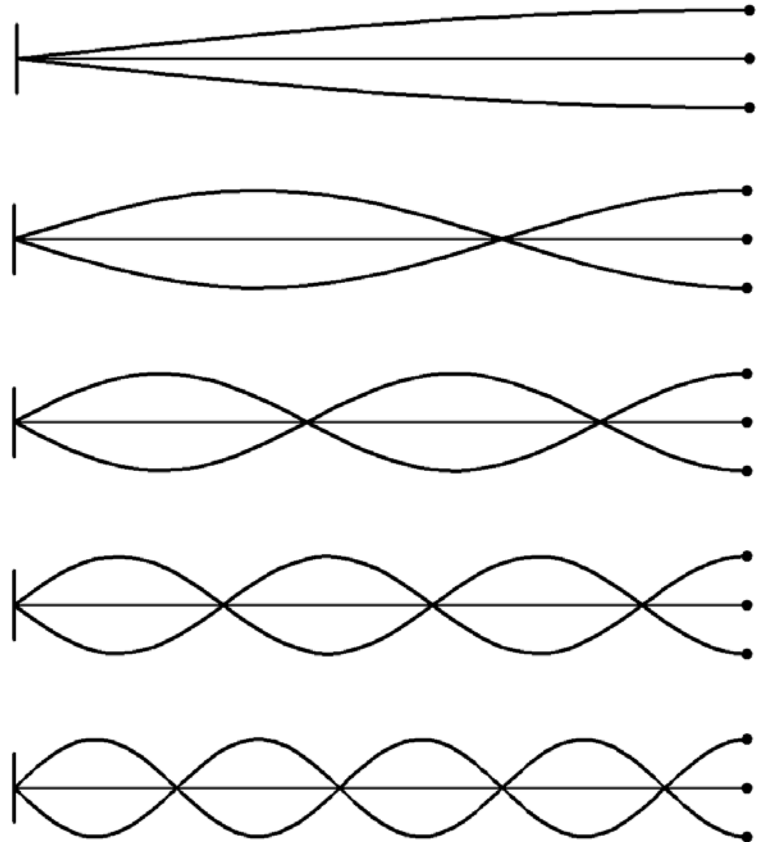
$$u_2 = u_0 \sin(\omega t + kx)$$

$$u = u_1 + u_2 = 2u_0 \cos kx \, \sin \omega t$$



nodes and antinodes (periodic compression and dilution)





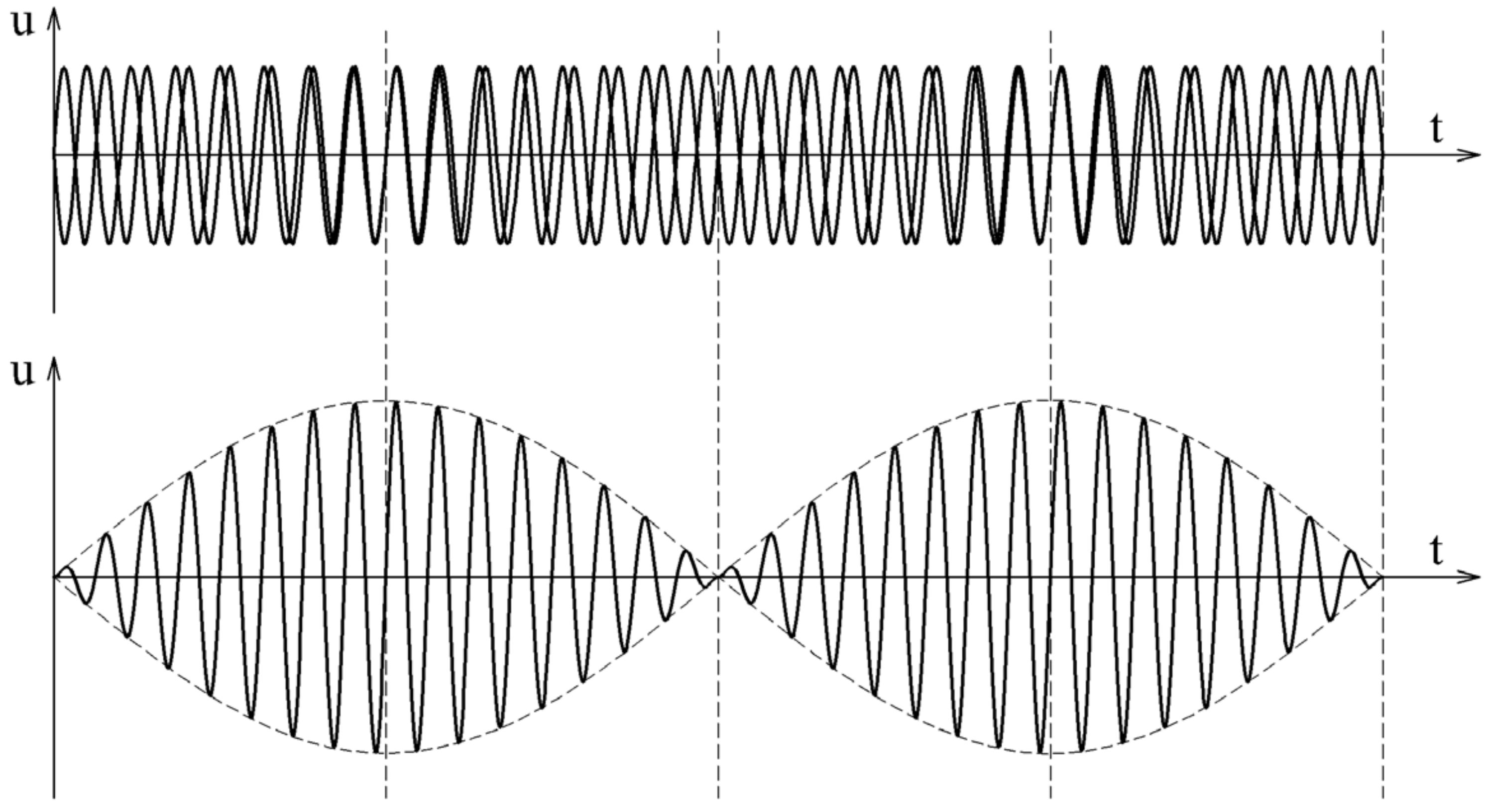
$$n\frac{\lambda}{2} = L \quad , \quad n = 1, 2, ..$$

$$f_n = n\frac{c}{2L} = nf_1 \quad , \quad n = 1, 2, 3, ...$$

$$n\frac{\lambda}{4} = L \quad , \quad n = 1,\ 3, 5,\ ...$$

$$f_n = nf_1 = n\frac{c}{4L}$$

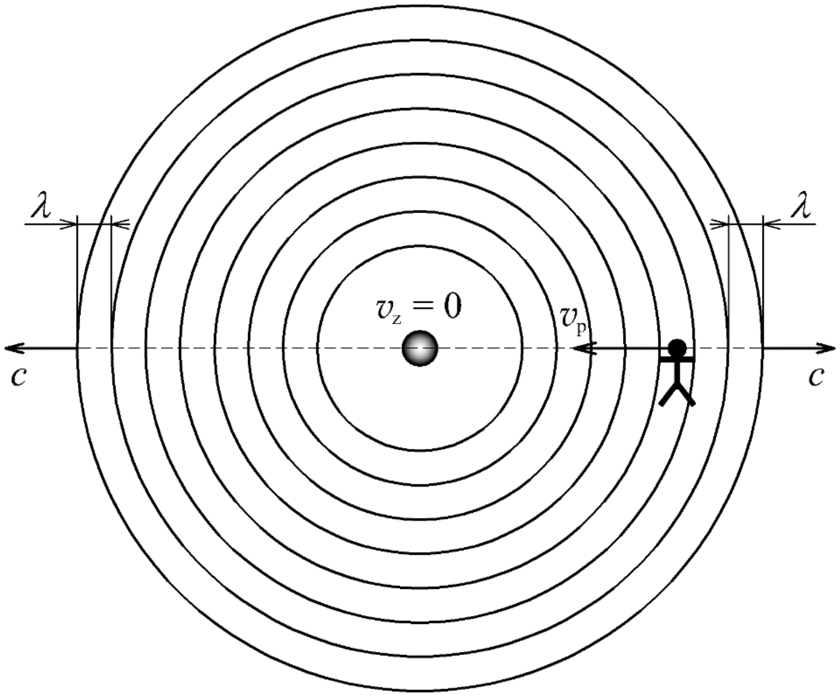
Interference of Similar Frequencies



beats $f_r = f_2 - f_1$

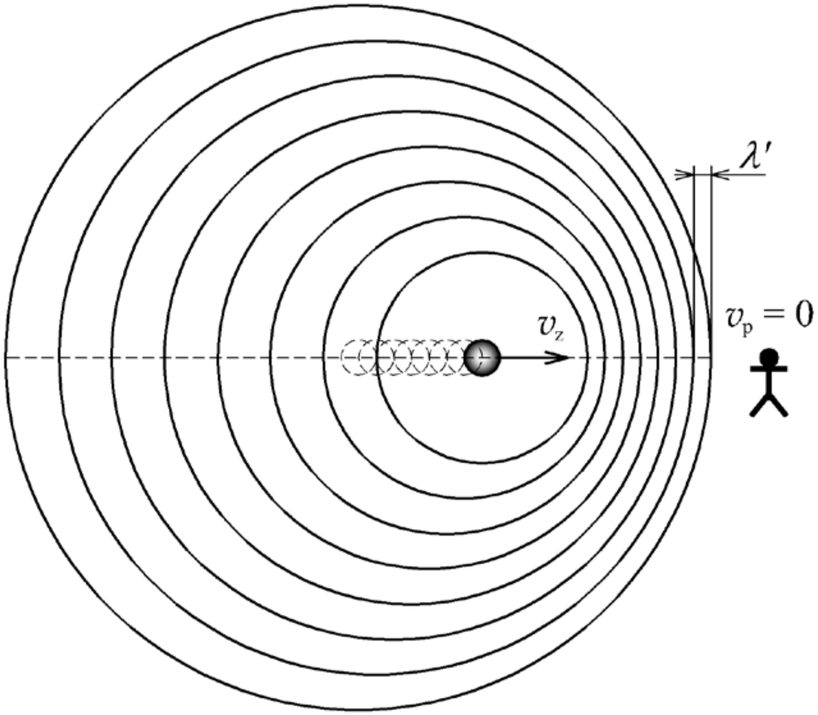
Doppler Effect

Observer's motion



wave front speed for the observer $c + v_p$

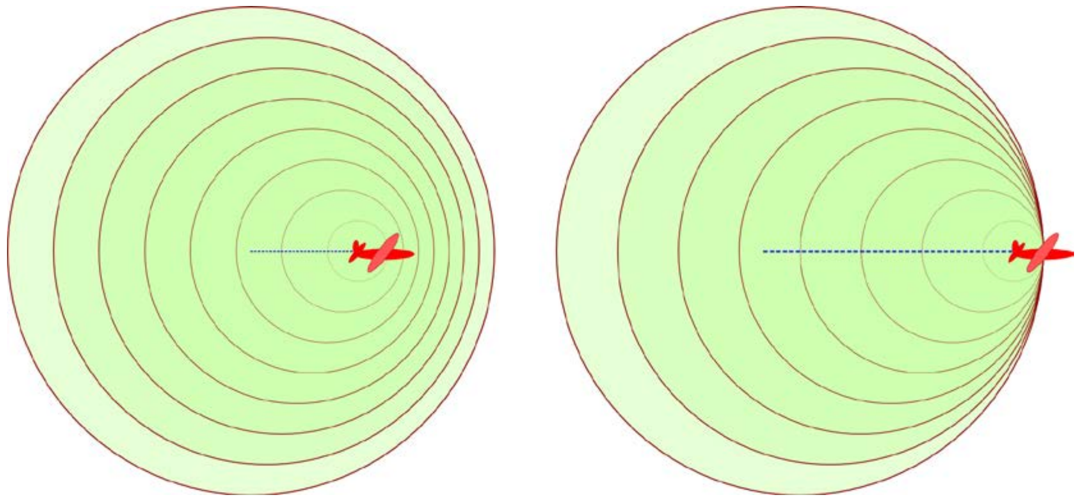
Source motion

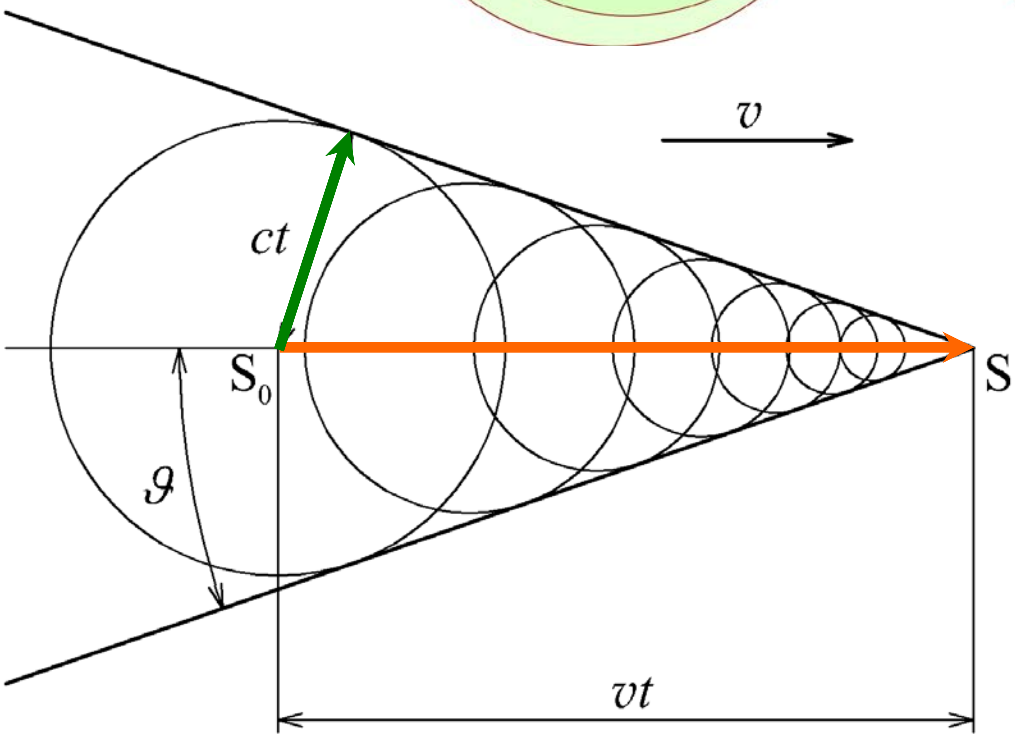


wavelength reduced due to source path $\lambda = \lambda_0 - T v_z$

$$f_p = \frac{c + v_p}{c} f_0 \quad \rightarrow \quad f_p = \frac{c + v_p}{c - v_z} f_0 \quad \leftarrow \quad f_p = \frac{c}{c - v_z} f_0$$

Shock Wave





$$\frac{v}{c}$$ Mach number

$$\sin \vartheta = \frac{c}{v}$$


LINE1
0:50:14

Energy and Intensity of Waves

$$I = \frac{1}{S}\frac{dW}{dt}$$ harmonic wave intensity

$$w = \frac{dW}{dV} \qquad \Delta W = wSc\Delta t \qquad I = wc$$

$$\Delta W = \frac{1}{2}\Delta m\,\omega^2 u_0^2$$

$$w = \frac{1}{2}\frac{\Delta m\,\omega^2 u_0^2}{\Delta V} = \frac{1}{2}\frac{\rho\,\Delta V\,\omega^2 u_0^2}{\Delta V}$$

$$w = \frac{1}{2}\rho\omega^2 u_0^2$$

$$I = \frac{1}{2}\rho\omega^2 c u_0^2$$

$$I = \frac{1}{2}v_0^2\rho c = \frac{1}{2}\frac{p_0^2}{\rho c}$$

Intensity level $L_I = 10\log\frac{I}{I_0}$

$I_0 = 1.10^{-12}$ W.m^{-2} threshold of hearing intensity