

## Calculation of the Enclosed Volumes and Surface Areas:

Measurement error of a calliper:  $m(x) = 0,1 \text{ mm}$

Measurement error of a micrometre  $m(x) = 0,01 \text{ mm}$

$$u_{xB} = \frac{m(x)}{\sqrt{3}}$$

### Sphere:

$$V = \frac{\pi d^3}{6}$$

$$S = \pi d^2$$

$$u_{rV} = 3u_{rd} \quad u_{rd} = \frac{u_d}{d}$$

$$u_{rS} = 2u_{rd}$$

### Cylinder:

$$V = \frac{\pi d^2 h}{4}$$

$$S = \frac{\pi d^2}{2} + \pi dh$$

$$u_{rV} = \sqrt{4u_{rd}^2 + u_{rh}^2} \quad u_{rd} = \frac{u_d}{d}, \quad u_{rh} = \frac{u_h}{h}$$

$$u_S = \sqrt{\left(\frac{\partial S}{\partial d}\right)^2 u_d^2 + \left(\frac{\partial S}{\partial h}\right)^2 u_h^2} = \pi \sqrt{(d+h)^2 u_d^2 + d^2 u_h^2}$$

### Cuboid:

$$V = abc$$

$$S = 2(ab + ac + bc)$$

$$u_{rV} = \sqrt{u_{ra}^2 + u_{rb}^2 + u_{rc}^2} \quad u_{ra} = \frac{u_a}{a} \quad u_{rb} = \frac{u_b}{b} \quad u_{rc} = \frac{u_c}{c}$$

$$u_S = \sqrt{\left(\frac{\partial S}{\partial a}\right)^2 u_a^2 + \left(\frac{\partial S}{\partial b}\right)^2 u_b^2 + \left(\frac{\partial S}{\partial c}\right)^2 u_c^2} = 2\sqrt{(b+c)^2 u_a^2 + (a+c)^2 u_b^2 + (a+b)^2 u_c^2}$$

## Swing time $\tau$ of a pendulum :

For our purpose, the period  $T$  should be recorded.

### One period as a single measurement:

$$u_T = \frac{0,5}{\sqrt{3}} \text{ s} \quad u_\tau = \frac{0,5}{2\sqrt{3}} \text{ s} \quad \text{Human reaction time}$$

### 10 periods as a single measurement $t = 10T = 20\tau$ :

$$u_t = \frac{0,5}{\sqrt{3}} \text{ s} \quad u_T = \frac{0,5}{10\sqrt{3}} \text{ s} \quad u_\tau = \frac{0,5}{20\sqrt{3}} \text{ s}$$

### Repeated measurement of one period $t = T = 2\tau$ :

$$\bar{t} = \frac{\sum_{i=1}^n t_i}{10}$$
$$s_{\bar{t}} = \sqrt{\frac{\sum_{i=1}^n (\bar{t} - t_i)^2}{10(10-1)}} = u_t \quad u_{\tau A} = \frac{u_t}{2} \quad u_{\tau B} = \frac{u_{tB}}{2} = \frac{0,01}{2\sqrt{3}} \text{ s} \quad \text{Stop-watch error}$$
$$u_\tau = \sqrt{u_{\tau A}^2 + u_{\tau B}^2}$$

### Sequential measurement of one period (10 data points) $t = T = 2\tau$ :

$$\text{Linear regression } y = ax \quad \tau = \frac{a}{2} \quad u_{\tau A} = \frac{u_t}{2} = \frac{s_a}{2}$$

$$u_{\tau B} = \frac{u_{tB}}{2} = \frac{0,01}{2\sqrt{3}} \text{ s}$$

$$u_\tau = \sqrt{u_{\tau A}^2 + u_{\tau B}^2}$$

### Computer controlled measurement:

Use the software "Torzni kyvadlo".

Consider the uncertainty  $u_{\tau A}$  only.