

Seminary exercise Nr. 6

Rigid body

1. A thin, uniform rod has a mass M and a length L . What is the moment of inertia of the rod about the axis being perpendicular to the rod length and passing through its centre of gravity?
2. A thin, uniform rod has a mass M and a length L . What is the moment of inertia of the rod about the axis being perpendicular to the rod length and passing through the end of the rod? Check the result using the parallel axis theorem and the solution of problem Nr.1.
3. Determine the moment of inertia about the central axis of a thin circular disc of mass M , diameter D and height H .
4. A physical pendulum consists of two parts: 1) a rod of mass 0.5 kg and length 95 cm and 2) a ball of mass 4 kg and diameter 10 cm located at the end of the rod. The rotational axis of the system is passing through the other end of the rod. What is the moment of inertia of the pendulum?
5. A helicopter rotor consists of three blades, each with length 7.80 m and mass 110 kg (suppose that the shape is a thin uniform rod, $I = \frac{1}{3}ML^2$). The blades are attached to the rotor axle by a single bolt. The rotor starts at rest and reaches the speed of 600 min^{-1} in 180 s . What is the average torque acting on the rotor?
6. A cylindrical flywheel ($I = \frac{1}{2}MR^2$) rotating about the central axis has a mass of 10 kg and a diameter of 20 cm . A constant torque acts on the flywheel at rest and, due to it, the flywheel reaches the frequency of 300 Hz after 30 s . What is the magnitude of the acting torque?
7. A gyroscope with momentum of inertia of 0.2 kgm^2 rotates at a constant speed of 1800 min^{-1} . Due to an acting torque, the speed of the gyroscope increases to 3000 min^{-1} within 2.4 s . Find the expression of the acting torque considering the fact that the torque is linearly decreasing from its maximal value at $t_0=0\text{ s}$ to zero at $t_1=2.4\text{ s}$.
8. A wheel rotates clockwise about its central axis with an angular momentum of $600\text{ kgm}^2\text{ s}^{-1}$. At the time $t_0=0$, a torque of magnitude 50 Nm is applied to the wheel to reverse the rotation. At what time t_1 is the angular speed zero?
9. A ball ($I = \frac{2}{5}MR^2$) of mass $m=50\text{ g}$ and diameter $d=3\text{ cm}$ rolls smoothly down an incline that leads into a cylindrical loop of radius $r=50\text{ cm}$. From what height h must the ball go down to pass through the entire circular loop of the cylindrical surface?
10. What is the kinetic energy of a hoop ($I = MR^2$) rolling smoothly on a horizontal path? The hoop diameter is 50 cm , its mass is 2 kg and the linear velocity of the hoop centre is 2 m s^{-1} .
11. A figure skater starts the spin with outstretched limbs. If she draws her limbs inward, she reduces her moment of inertia considerably and the spin speed increases. Her initial moment of inertia is I_0 and, when she draws her limbs in, her moment of inertia decreases by 10% . What is the rate of the initial and final angular velocity? What is the change of the kinetic energy?