Experiment Nr. 26

DETERMINATION OF THE ACOUSTIC WAVE SPEED
AND YOUNG'S MODULUS OF SOLID BODY

Theoretical part

Acoustic waves (sound) can propagate within a material substance called a medium (solid, liquid, gaseous state and plasma). A sound wave is a travelling longitudinal elastic wave, thus, it causes a periodic compression and dilution of the medium. Near a point source, the wave-fronts are spherical and are spreading out in three dimensions (spherical waves). As the wave-fronts move outward and their radii become larger, their curvatures decrease. Far from the source they can be assumed to be planar.

The human ear is sensitive to sound waves in the frequency range from about 16 Hz to 20 kHz. The sound waves with frequencies higher than 20 kHz are called as ultrasound. The sound waves with frequencies lower than 16 Hz are called as infrasound.

The simplest sound waves are sinusoidal waves with definite frequency, amplitude and wavelength. A sinusoidal sound wave in an elastic medium is described by the following wave function

$$u = u_0 \sin \omega \left(t - \frac{x}{c} \right)$$

where u_0 is the wave amplitude, ω is the angular velocity, t is time and c is the phase velocity of the wave in the selected medium. The wavelength λ and the wave frequency f define the phase velocity by following formula

$$\lambda = \frac{c}{f}$$

Sound waves can also be described in terms of the air pressure variation. The pressure fluctuates above and below the atmospheric pressure. Its variation in time is sinusoidal and has the same angular frequency ω as that of the motion of air particles. The sound pressure variations are given by

$$p = p_0 \cos \omega (t - \frac{x}{c}) \, , \, [p] = \text{Pa}$$

Thus the relation between the pressure and the sound wave function is

$$p = \rho v c$$

where ρ is the medium density and v is the sound particle velocity (it's the velocity of the wave oscillations, not the phase velocity !). The pressure amplitude for the lowest detectable sound intensity at the frequency 1 kHz is about $2.8 \cdot 10^{-5}$ Pa. The root-mean-square (RMS or effective) value of the pressure is

$$p_{ef} = \frac{p_0}{\sqrt{2}}$$

The maximum pressure amplitude p_0 of a sound wave that the human ear can tolerate is about 28 Pa (much less than the normal atmospheric pressure of about 10^5 Pa).

The intensity I of a travelling wave is defined as the time average rate at which energy is transported by the wave, per unit area, across a surface perpendicular to the direction of propagation. Briefly, the intensity is the average power transported per unit area that is perpendicular to the direction of the wave propagation. The intensity of as sound wave can be expressed as

$$I = \frac{p_{ef}^2}{\rho c}, \quad [I] = W \cdot m^{-2}$$

where the speed of sound c is temperature sensitive according to the relation (temperature t is in °C and speed in m.s^{-1})

$$c = 344.3 + 0.62\,(t - 20)$$

The sound wave phase speed can be determined directly by the time-lag sound pulse measurement on a selected path. More commonly, the sound wave speed is determined from the sound wavelength and frequency determination. According to the wave theory, the sound phase speed in a selected environment is

$$c = \sqrt{\frac{K}{\rho}} \ \text{ or } \ c = \sqrt{\frac{E}{\rho}},$$

where K is the bulk modulus (compressibility factor) for gases and liquids, E is the Young's modulus for solids and ρ is the density of the environment (gaseous, liquid or solid state).

The determination of the sound wave speed using Kundt's tube is based on the standing waves generated in the air column by an appropriate mechanical or electrical system. The stationary waves can be visualized by a liquid or a powder placed inside the tube.

The sound wave propagation in a solid material is based on the Hooke's law theory where the acting force responsible for the local displacement is caused by the elastic wave. This approach gives an opportunity to determine the Young's modulus of the material which the longitudinal acoustic wave is propagating in.

The Young's moduli of selected materials are arranged in the following table:

material	E [Pa]
steel	$2{,}1 \cdot 10^{11}$
aluminum	$(6{,}6\text{–}6{,}8) \cdot 10^{10}$
copper	$(1{,}2\text{–}1{,}3) \cdot 10^{11}$
brass	$(1{,}0\text{–}1{,}7) \cdot 10^{11}$
duralumin	$7{,}2 \cdot 10^{10}$

Theory of the measurement principle

The phase speed of sound correspond to the velocity of the longitudinal wave propagating in the specific environment. The phase speed is defined by the wave properties as

$$c = \lambda f \, ,$$

where λ is the wavelength and f wave frequency.

The speed of sound in a particular metal rod can be easily determined by means of so called Kundt's tube. A schematic plot of the measurement principle could be seen in Fig. 1.
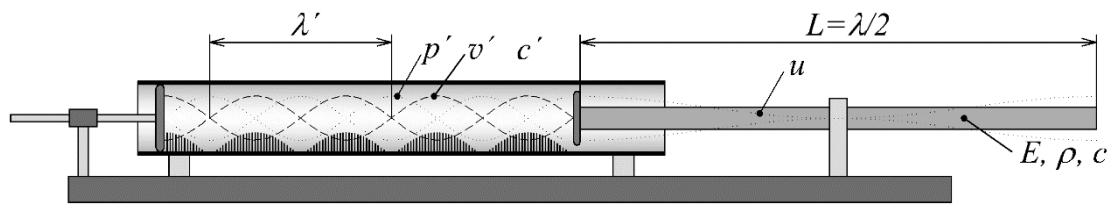

Fig. 1 A schematic plot of the Kundt's tube

A metal rod is fixed in its centre (on the right in Fig. 1) allowing a generation of the longitudinal mechanical waves that are transmitted to the air column within the glass tube (on the left in Fig. 1). The waves transmitted to the air column are reflected at the adjustable plug. In case the total length of the air column correspond to the odd $\frac{\lambda'}{4}$ -multiple, the standing waves would be generated within the glass tube. Inside the glass tube there is a small amount of uniformly distributed cork powder. When the standing waves are generated, the cork particles follow the antinodes of the acoustic velocity and, thus, the standing waves in the air column become visible. The longitudinal waves generated in the rod can have different frequencies, however, there is a strongest wave having the fundamental frequency. The corresponding fundamental wavelength is

$$\lambda = 2L \, ,$$

where L is the length of the metal rod (Fig. 1). In the metal rod, the phase speed of the sound waves can be defined as

$$c = 2Lf$$

We cannot determine the frequency f directly. First we will examine the wavelength λ' in the air column that could be directly observed from the cork powder spread. We will record several positions of the standing wave nodes to get an average distance between nodes that correspond to the value of $\frac{\lambda'}{2}$.

Considering the same sound wave frequency in the rod as well as in the air column $f = f'$, we can conclude that

$$c = c' \frac{\lambda}{\lambda'} \, ,$$

where c is the phase speed of sound in the metal rod and c' is the phase speed of the sound wave

in the air.

As afore-mentioned, the theoretical phase speed of the sound wave in a selected solid material is

$$c = \sqrt{\frac{E}{\rho}}\ ,$$

where the E is the metal rod Young's modulus and ρ is its material density. Thus, using the previous definition we get a formula for E determination:

$$E = \rho\left(c'\frac{2L}{\lambda'}\right)^2$$

Note that the phase speed of the sound wave in the air should be determined with respect to the ambient air temperature due to better results accuracy. Use the formula

$$c' = 344.3 + 0.62\,(t\text{ - }20),$$

where t is the ambient air temperature in degrees of Celsius.

Measurement objectives

1. Determine the wavelength λ' of the sound wave in the air column.

2. Calculate the phase speed of the sound wave in the air column c'.

2. Determine the Young's modulus of the metal rod and evaluate the measurement uncertainty. Compare the obtained result to the material property given by the tabular data.

Measurement procedure

First, try to generate the longitudinal mechanical waves by rubbing metal rod with a deerskin leather. If the air column length is well-set, the waves transmitted to the air column will form the standing waves. Follow the nodes spacing (see Fig. 1) and read about 7 - 10 positions of the nodes using a tape measure (start from the second or third node from the adjustable plug). For higher result accuracy, use the regression of the linear function $x_i = f(i)$, where x_i is the particular position of a node and i is its number of order. Note, that the wavelength $\lambda' = 2a$, where a is the slope coefficient of the afore-mentioned linear function.

The fundamental wavelength of the sound waves in the metal rod correspond to the rod length L. Determine this value using a tape measure.

Important constants

The metal rod is made of copper with density $\rho_{Cu} = (8960 \pm 10)$ kg.m^{-3}.

Uncertainty calculation notes

Phase speed in the metal rod c
The combined uncertainty consist of both Type A and Type B uncertainty. The relative Type A uncertainty can be determined by

$$u_{cA} = c' \frac{\lambda}{\lambda'^2} u_{\lambda'A},$$

where $u_{\lambda'A} = 2u_{aA}$ because $\lambda' = 2a$ (see above). The slope coefficient uncertainty u_{aA} is given by the regression calculation results.
The relative Type B uncertainty can be calculated from the Type B sub uncertainty of the wavelength λ' measurement (uncertainty of c' can be neglected) as follows:

$$u_{cB} = \frac{c'}{\lambda'} u_{\lambda B}$$

The combined standard uncertainty of the phase speed is

$$u_c = \sqrt{u_{cA}^2 + u_{cB}^2}$$

Young's modulus E of the metal rod
The combined uncertainty consist of the Type B uncertainty only. The value can be determined by

$$u_{rEB} = \sqrt{u_{r\rho B}^2 + 4u_{rcB}^2} \,,$$

where $u_{r\rho B}$ is the relative uncertainty of the rod material density (see the constants above).