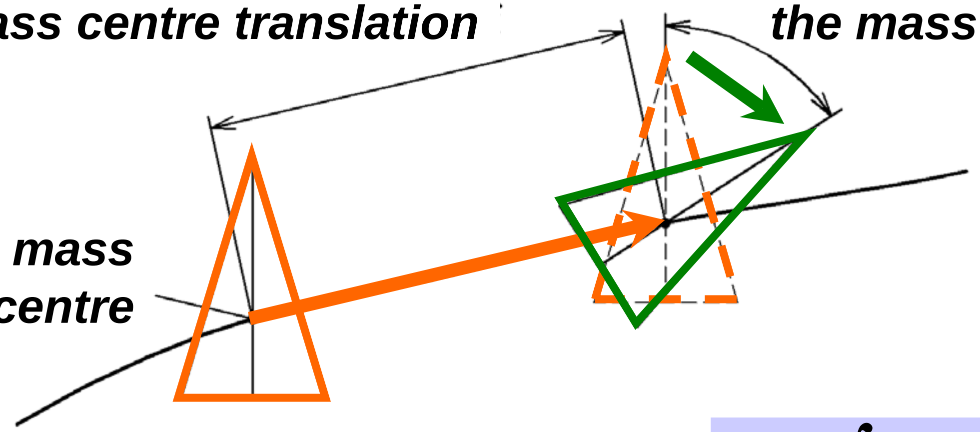
Rigid body

fixed positions of the mass particles (also during the motion)

6x DoF !!

rigid body motion = **translation** of the mass centre
+ **rotation** about the axis passing the mass centre



$$dm = \rho \, dV$$

$$m = \int_{(m)} dm = \int_{(V)} \rho \, dV$$

Centre of gravity

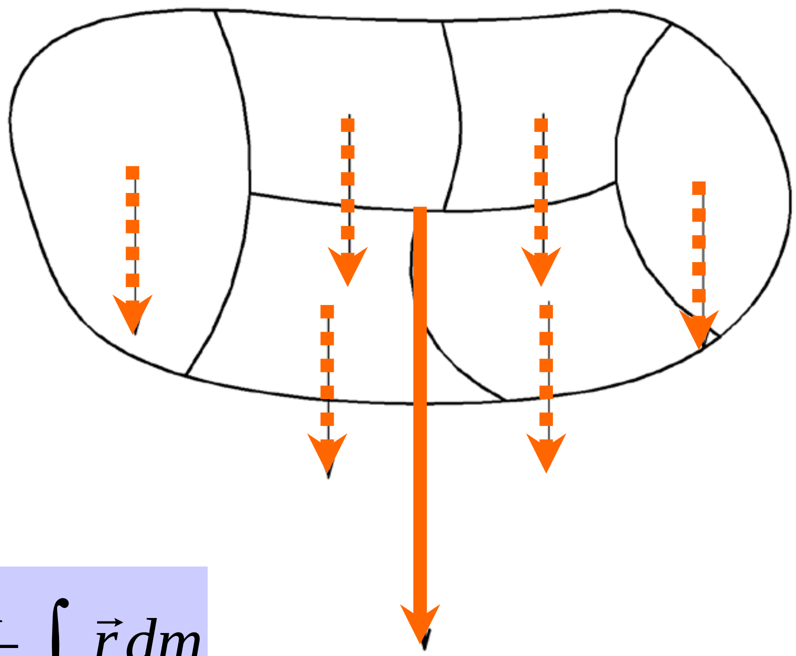
point of acting force of gravity; any torque equals zero

$$\vec{F}_g = \sum_i m_i \vec{g}$$

$$\vec{r}_T \times \vec{F}_g = \sum_i \left(\vec{r}_i \times m_i \vec{g} \right)$$

$$\vec{r}_T m \times \vec{g} = \sum_i \left(\vec{r}_i m_i \times \vec{g} \right)$$

$$\vec{r}_T = \frac{1}{m} \sum_i \vec{r}_i m_i$$



$$\vec{r}_T = \frac{1}{m} \int_{(m)} \vec{r}\, dm$$

centre of gravity – homogenous bodies in homogenous field

$$\vec{r}_T = \vec{r}_s$$

Rigid body dynamics

Translation

$$\vec{F} = m\frac{d^2\vec{r}_s}{dt^2} = \frac{d\vec{p}}{dt}$$

force and linear momentum equivalent

Rotation about a fixed axis

Torque

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{M} = \vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt}$$

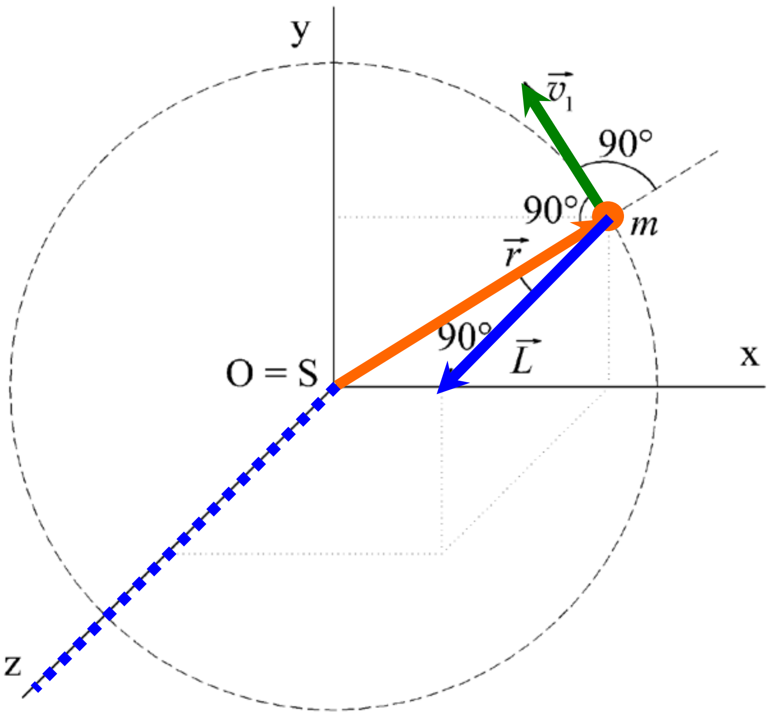
Angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}$$

$$\vec{M} = \frac{d\vec{L}}{dt}$$



$$\left|\vec{L}\right| = \left|\vec{r}\times\vec{p}\right| = \left|\vec{r}\times m\vec{v}\right| = rmv = mr^2\omega$$

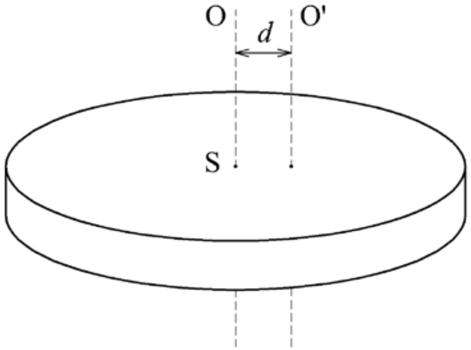
$$\vec{L} = \sum_i \vec{L}_i$$ for particle system

$$\vec{L}_i = m_i r_i^2 \vec{\omega}$$

$$\vec{L} = \sum_i \vec{L}_i = \left(\sum_i m_i r_i^2\right)\vec{\omega}$$

$$\sum_i m_i r_i^2 = I \qquad \vec{L} = I\vec{\omega}$$

Moment of Inertia $$I = \int_{(m)} r^2 dm$$



$$I = I_0 + md^2$$ **Steiner's theorem**

$$\vec{M} = \sum_i \vec{M}_i = \sum_i \frac{d\vec{L}_i}{dt} = \frac{d}{dt}\left(\sum_i \vec{L}_i\right) = \frac{d\vec{L}}{dt}$$

$$\vec{M} = \frac{d\vec{L}}{dt} = \frac{d(I\vec{\omega})}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\varepsilon}$$

$$\vec{M} = \sum_i \vec{M}_i = \frac{d\vec{L}}{dt}$$

Torque and angular momentum equivalent

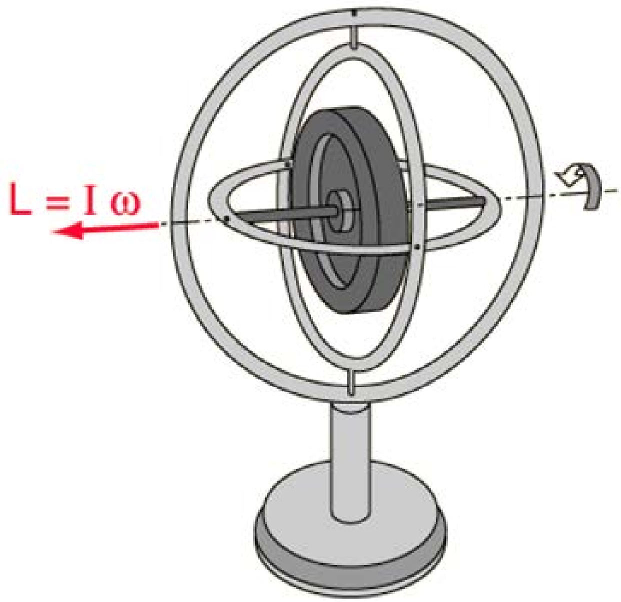
$$\vec{M} = \sum_i \vec{M}_i = 0 = \frac{d\vec{L}}{dt} \Rightarrow \vec{L} = \overrightarrow{\text{konst}}$$

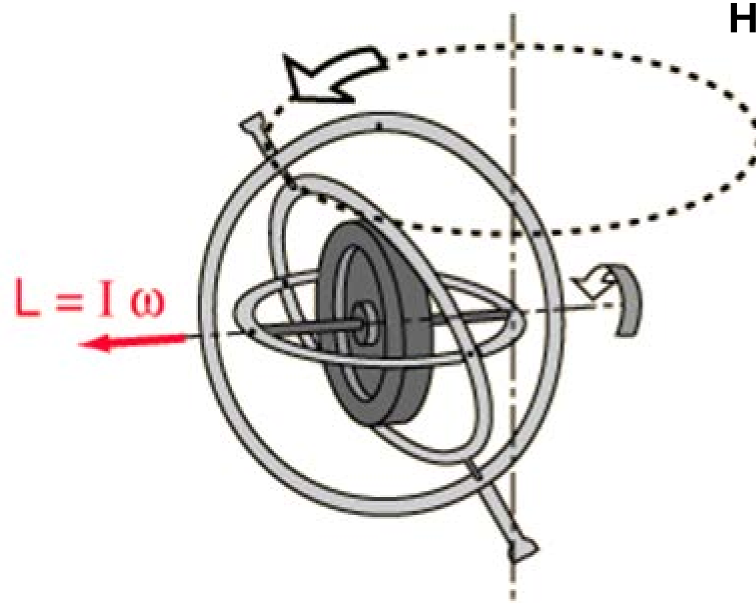
Conservation of angular momentum (CAM)

Static equilibrium of rigid body

$$\sum_i \vec{F}_i = 0$$

$$\sum_i \vec{M}_i = \sum\left(\vec{r}_i \times \vec{F}_i\right) = 0$$


L = I ω


L = I ω


Fest
Los

Rotational kinetic energy of a rigid body

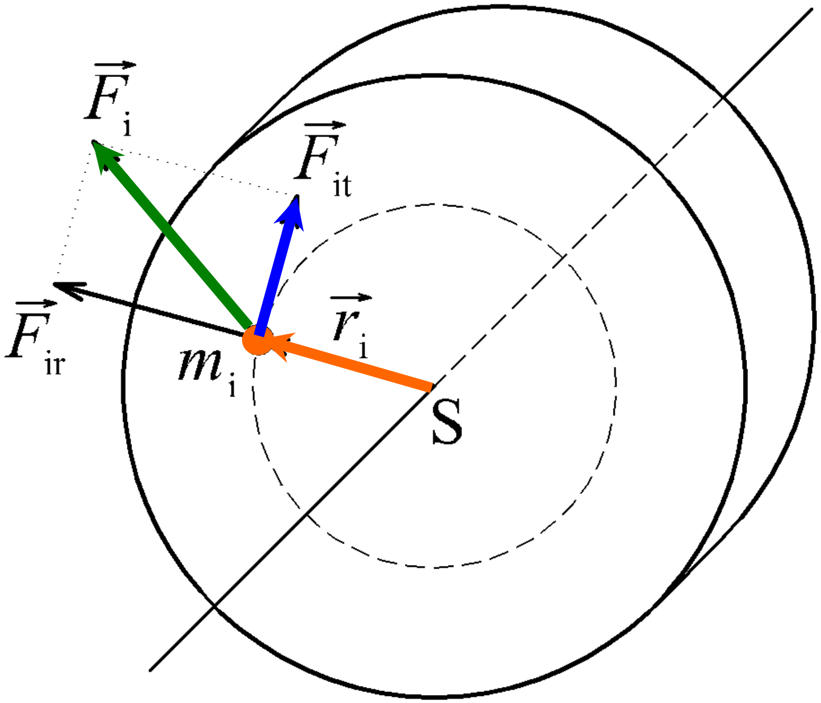
$$ds_i = r_i \, d\varphi$$

$$dA_i = F_{it} \, ds_i = M_i \, d\varphi$$

$$dA = M \, d\varphi$$

$$P = \frac{dA}{dt} = M \omega$$

$$P = \vec{M} \cdot \vec{\omega}$$



$$W_k = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2} I \omega^2$$

$$W_k = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$