Seminary exercise Nr. 2
Field of gravitation

For all exercises, let assume the following values:

- **gravitational constant** $G = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$
- **Earth radius** $r_E = 6.37 \cdot 10^6 m$
- **Earth mass** $m_E = 5.97 \cdot 10^{24} kg$

1. What is the intensity of the gravitational field at the Earth's surface?

$\|\vec{g}\|_E = ?$

$$\vec{g} = -G \frac{M}{r^2} \vec{u}_r \quad ; \quad \|\vec{g}\| = G \frac{M}{r^2}$$

$$\|\vec{g}\|_E = G \frac{m_E}{r_E^2} = \frac{6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2} \cdot 5.97 \cdot 10^{24} kg}{\left(6.37 \cdot 10^6 m\right)^2} = 9.81 m s^{-2}$$

2. At what altitude above the Earth's surface would the gravitational acceleration be $4.6 m s^{-2}$?

$\|\vec{g}\| = 4.6 m s^{-2}$

$h = ?$

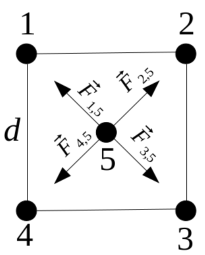
$$\|\vec{g}\| = \frac{G m_E}{\left(r_E + h\right)^2}$$

$$h = \sqrt{\frac{G m_E}{\|\vec{g}\|}} - r_E = \sqrt{\frac{6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2} \cdot 5.97 \cdot 10^{24} kg}{4.6 m s^{-2}}} - 6.37 \cdot 10^6 m = 2.93 \cdot 10^6 m$$

3. Four mass particles, each of mass m, form a square with an edge length of d. What gravitational force (magnitude and direction) acts to the fifth mass particle of mass m placed in the center of the square? What is the gravitational field intensity at the same point?

m

d



$$\vec{F}_{1,5} = -\vec{F}_{3,5} \quad ; \quad \vec{F}_{2,5} = -\vec{F}_{4,5} \quad ; \quad \vec{F}_5 = \sum_{i=1}^{4} \vec{F}_{i,5} = 0 \quad ; \quad \vec{g}_5 = 0$$

4. A space rocket is in a circular orbit at the altitude of 400 km above the Earth's surface. What is the magnitude of the centripetal acceleration acting to the rocket? At what speed it is moving? What is the period of the orbit?

$h = 400 km = 4 \cdot 10^5 m$

$\|\vec{a}\| = ?$

$v = ?$

$T = ?$

$$\|\vec{F}\| = m g(r) = G \frac{m M}{r^2}$$

$$\|\vec{a}\| = g(r_E + h) = \frac{G m_E}{(r_E + h)^2} = \frac{6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2} \cdot 5.97 \cdot 10^{24} kg}{(6.37 \cdot 10^6 m + 4 \cdot 10^5 m)^2} = 8.68 m s^{-2}$$

$$\|\vec{a}\| = \frac{v^2}{r} \quad ; \quad v = \sqrt{\|\vec{a}\|(r_E + h)} = \sqrt{8.68 m s^{-2} \cdot (6.37 \cdot 10^6 m + 4 \cdot 10^5 m)} = 7670 m s^{-1}$$

$$v = \frac{2\pi \cdot (r_E + h)}{T} \quad ; \quad T = \frac{2\pi \cdot (r_E + h)}{v} = \frac{2\pi \cdot (6.37 \cdot 10^6 m + 4 \cdot 10^5 m)}{7670 m s^{-1}} = 5550 s$$

10. Determine the orbit altitude for a geostationary satellite. (It is also called a fixed satellite, because its position is fixed above the same place on the Earth's surface)

$T = 24\,h =$
$= 86400\,s$

$h = ?$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86400\,s} = 7.27\cdot 10^{-5}\,rad\,s^{-1}$$

$$a(r_E + h) = \frac{G m_E}{(r_E + h)^2} = \omega^2 \cdot (r_E + h) \quad ; \quad (r_E + h)^3 = \frac{G m_E}{\omega^2}$$

$$h = \sqrt[3]{\frac{G m_E}{\omega^2}} - r_E = \sqrt[3]{\frac{6.67\cdot 10^{-11}\,m^3 kg^{-1} s^{-2} \cdot 5.97\cdot 10^{24}\,kg}{(7.27\cdot 10^{-5}\,s^{-1})^2}} - 6.37\cdot 10^6\,m = 3.59\cdot 10^7\,m$$

12. What is the escape speed on the Earth?

$v_e = ?$

On Earth: $U_E = -\frac{G m_E M}{r_E}$; $K_E = \frac{1}{2} M v_e^2$

At infinite distance: $U_\infty = 0$; $K_\infty = 0$

$$U_E + K_E = U_\infty + K_\infty = 0 \quad ; \quad \frac{1}{2} M v_e^2 = \frac{G m_E M}{r_E}$$

$$v_e = \sqrt{\frac{2 G m_E}{r_E}} = \sqrt{\frac{2\cdot 6.67\cdot 10^{-11}\,m^3 kg^{-1} s^{-2} \cdot 5.97\cdot 10^{24}\,kg}{6.37\cdot 10^6\,m}} = 1.12\cdot 10^4\,m\,s^{-1}$$