Seminary exercise Nr. 5

Systems of particles, collisions

1. A $1.2\,kg$ ball drops vertically onto the floor, hitting it with a speed of $25\,m\,s^{-1}$. The ball bounces back with an initial speed of $10\,m\,s^{-1}$. What impulse acts on the ball during the contact? If the ball is in contact with the floor for $0.02\,s$, what is the magnitude of the average force on the floor from the ball?

$m=1.2\,kg$

$v_b=-25\,m\,s^{-1}$

$v_a=10\,m\,s^{-1}$

$J=?$

$\Delta t=0.02\,s$

$\bar{F}=?$

$$\vec{F}=\frac{d\vec{p}}{dt} \quad ; \quad \vec{J}=\int_{t_1}^{t_2}\vec{F}\,dt=\int_{t_1}^{t_2}\frac{d\vec{p}}{dt}dt=\int_{t_1}^{t_2}d\vec{p}=\vec{p}(t_2)-\vec{p}(t_1)=\Delta\vec{p}$$

$$J=\Delta p=p_a-p_b=m(v_a-v_b)=1.2\,kg\cdot(10\,m\,s^{-1}+25\,m\,s^{-1})=42\,kg\,m\,s^{-1}$$

$$\bar{F}=\frac{J}{\Delta t}=\frac{42\,kg\,m\,s^{-1}}{0.02\,s}=2100\,N$$

5. A cart with mass $340\,g$ moving on a frictionless linear track at an initial speed of $1.2\,m\,s^{-1}$ undergoes an elastic collision with an initially stationary cart of unknown mass. After the collision, the first cart continues in its original direction at $0.66\,m\,s^{-1}$. What is the mass of the second cart? What is its speed after the impact? What is the speed of the two-cart centre of mass?

$m_1=340\,g=$
$=0.34\,kg$

$v_{1,b}=1.2\,m\,s^{-1}$

$v_{2,b}=0$

$v_{1,a}=0.66\,m\,s^{-1}$

elastic collision

$m_2=?$

$v_{2,a}=?$

$v_{CoM,a}=?$

$$\begin{cases} m_1 v_{1,b}+m_2 v_{2,b}=m_1 v_{1,a}+m_2 v_{2,a} \\ \frac{1}{2}m_1 v_{1,b}^2+\frac{1}{2}m_2 v_{2,b}^2=\frac{1}{2}m_1 v_{1,a}^2+\frac{1}{2}m_2 v_{2,a}^2 \end{cases} \quad ; \quad v_{2,a}=\frac{m_1}{m_2}(v_{1,b}-v_{1,a})$$

$$m_1 v_{1,b}^2=m_1 v_{1,a}^2+m_2\frac{m_1^2}{m_2^2}(v_{1,b}-v_{1,a})^2 \quad ; \quad m_1(v_{1,b}^2-v_{1,a}^2)=\frac{m_1^2}{m_2}(v_{1,b}-v_{1,a})^2$$

$$m_2=m_1\frac{(v_{1,b}-v_{1,a})^2}{v_{1,b}^2-v_{1,a}^2}=m_1\frac{v_{1,b}-v_{1,a}}{v_{1,b}+v_{1,a}}=0.34\,kg\,\frac{1.2\,m\,s^{-1}-0.66\,m\,s^{-1}}{1.2\,m\,s^{-1}+0.66\,m\,s^{-1}}=0.0987\,kg$$

$$v_{2,a}=\frac{m_1}{m_2}(v_{1,b}-v_{1,a})=\frac{0.34\,kg}{0.0987\,kg}(1.2\,m\,s^{-1}-0.66\,m\,s^{-1})=1.86\,m\,s^{-1}$$

$$v_{CoM,b}=\frac{m_1 v_{1,b}+m_2 v_{2,b}}{m_1+m_2}=\frac{m_1}{m_1+m_2}v_{1,b}=\frac{0.34\,kg}{0.34\,kg+0.0987\,kg}1.2\,m\,s^{-1}=0.930\,m\,s^{-1}$$

$$v_{CoM,a}=\frac{m_1 v_{1,a}+m_2 v_{2,a}}{m_1+m_2}=\frac{0.34\,kg\cdot 0.66\,m\,s^{-1}+0.0987\,kg\cdot 1.86\,m\,s^{-1}}{0.34\,kg+0.0987\,kg}=0.930\,m\,s^{-1}$$

6. A bullet of mass $10\,g$ strikes a ballistic pendulum of mass $2\,kg$. The centre of mass of the pendulum rises a vertical distance of $12\,cm$. Assuming that the bullet remains embedded in the pendulum, calculate the initial speed of the bullet.

$m=10\,g=$
$=0.01\,kg$
$M=2\,kg$
$h=12\,cm=$
$=0.12\,m$
$g=9.81\,m\,s^{-2}$
$v_0=?$

t_0 : instant before the bullet hits the pendulum

t_1 : instant after the bullet hits the pendulum

t_2 : instant when the pendulum reaches the height h

$$K_1=\frac{1}{2}(m+M)v_1^2 \quad ; \quad U_1=0 \quad ; \quad K_2=0 \quad ; \quad U_2=(m+M)g\,h$$

$$K_1+U_1=K_2+U_2 \quad ; \quad \frac{1}{2}(m+M)v_1^2=(m+M)g\,h$$

$$v_1=\sqrt{2\,g\,h}=\sqrt{2\cdot 9.81\,m\,s^{-2}\cdot 0.12\,m}=1.53\,m\,s^{-1}$$

$$m\,v_0=(m+M)v_1 \quad ; \quad v_0=\frac{m+M}{m}v_1=\frac{0.01\,kg+2\,kg}{0.01\,kg}1.53\,m\,s^{-1}=308\,m\,s^{-1}$$

8. A skater of mass $70\,kg$ stands on glassy ice. He puts himself in motion by firing horizontally a ball of mass $3\,kg$ at a speed of $8\,m\,s^{-1}$. How far will the skater move after firing the ball? The coefficient of kinetic friction between the ice and the skates is 0.02.

$m_s=70\,kg$
$m_b=3\,kg$
$v_b=8\,m\,s^{-1}$
$\mu_k=0.02$
$g=9.81\,m\,s^{-2}$
$d=?$

$$m_b v_b+m_s v_s=0 \quad ; \quad v_s=-\frac{m_b v_b}{m_s}=-\frac{3\,kg\cdot 8\,m\,s^{-1}}{70\,kg}=-0.34\,m\,s^{-1}$$

$$F_f=\mu_k m_s g \quad ; \quad a=\frac{F_f}{m_s}=\mu_k g=0.02\cdot 9.81\,m\,s^{-2}=0.196\,m\,s^{-2}$$

$$0-v_s=at \quad ; \quad t=-\frac{v_s}{a}=\frac{0.34\,m\,s^{-1}}{0.196\,m\,s^{-2}}=1.73\,s$$

$$-d-0=v_s t+\frac{1}{2}a t^2$$

$$d=-v_s t-\frac{1}{2}a t^2=0.34\,m\,s^{-1}\cdot 1.73\,s-\frac{1}{2}\cdot 0.196\,m\,s^{-2}(1.73\,s)^2=0.295\,m$$