Seminary exercise Nr. 9
Waves

2. The equation of a transverse wave traveling along a very long string is $y=6\sin(0.020\pi x-4\pi t)$, where x and y are expressed in centimeters and t is in seconds. Determine the amplitude, the wavelength, the frequency, the phase velocity and the maximum transverse velocity of a particle in the string. What is the transverse displacement at $x=3.5\,cm$ when $t=0.26\,s$?

$y=6\sin(0.020\pi x-4\pi t)$

$A=?$

$\lambda=?$

$f=?$

$v_p=?$

$v_{max}=?$

$x=3.5\,cm$

$t=0.26\,s$

$y=?$

$$A=6\,cm=6\cdot10^{-2}\,m \quad ; \quad \frac{2\pi}{\lambda}=0.020\,cm^{-1}\pi \quad ; \quad \lambda=\frac{2}{0.020\,cm^{-1}}=100\,cm=1\,m$$

$$f=\frac{\omega}{2\pi}=\frac{4\,s^{-1}\pi}{2\pi}=2\,Hz \quad ; \quad v_p=\lambda f=1\,m\cdot2\,Hz=2\,m\,s^{-1}$$

$$v_y=\frac{\partial y}{\partial t}=\frac{\partial}{\partial t}\left[6\sin(0.020\pi x-4\pi t)\right]=6\cos(0.020\pi x-4\pi t)(-4\pi)=$$

$$=-24\pi\cos(0.020\pi x-4\pi t)$$

$$v_{max}=24\,cm\,s^{-1}\cdot\pi=75.4\,cm\,s^{-1}=0.754\,m\,s^{-1}$$

$$y=6\sin(0.020\pi x-4\pi t)=6\sin(0.020\pi\cdot3.5-4\pi\cdot0.26)=-18.3\,cm=-0.183\,m$$

3. Determine the phase velocity for the longitudinal and transversal waves in a steel rod. The material constants of steel are $E=210\,GPa$, $G=80\,GPa$, $\rho=7850\,kg\,m^{-3}$.

$E=210\,GPa$

$G=80\,GPa$

$\rho=7850\,kg\,m^{-3}$

$v_p^l=?$

$v_p^t=?$

$$v_p^l=\sqrt{\frac{E}{\rho}}=\sqrt{\frac{210\cdot10^9\,Pa}{7850\,kg\,m^{-3}}}=5170\,m\,s^{-1} \quad ; \quad v_p^t=\sqrt{\frac{G}{\rho}}=\sqrt{\frac{80\cdot10^9\,Pa}{7850\,kg\,m^{-3}}}=3190\,m\,s^{-1}$$

5. A string of length $120\,cm$ is stretched between fixed supports. What are the longest, second longest, and third longest wavelengths for waves traveling on the string if standing waves are to be set up? What is the node-to-node distance for these wavelengths?

$l=120\,cm=$
$=1.2\,m$

$\lambda_1=?$

$\lambda_2=?$

$\lambda_3=?$

$d_1=?$

$d_2=?$

$d_3=?$

$$n\frac{\lambda_n}{2}=l \quad ; \quad \lambda_n=\frac{2l}{n}$$

$$\lambda_1=\frac{2l}{1}=2\cdot1.2\,m=2.4\,m \quad ; \quad d_1=l=1.2\,m$$

$$\lambda_2=\frac{2l}{2}=\frac{2\cdot1.2\,m}{2}=1.2\,m \quad ; \quad d_2=\frac{l}{2}=\frac{1.2\,m}{2}=0.6\,m$$

$$\lambda_3=\frac{2l}{3}=\frac{2\cdot1.2\,m}{3}=0.8\,m \quad ; \quad d_2=\frac{l}{3}=\frac{1.2\,m}{3}=0.4\,m$$

7. An ambulance with a siren emitting a whine at $1600\,Hz$ overtakes and passes a cyclist riding a bike at $2.44\,m\,s^{-1}$. After being passed, the cyclist hears a frequency of $1590\,Hz$. How fast is the ambulance moving? The sound speed in air is $344\,m\,s^{-1}$.

$f_0 = 1600\,Hz$

$v_c = 2.44\,m\,s^{-1}$

$f = 1590\,Hz$

$v_p = 344\,m\,s^{-1}$

$v_a = ?$

$$f = \frac{v_p + v_c}{v_p + v_a} f_0 \quad ; \quad v_p + v_a = (v_p + v_c)\frac{f_0}{f}$$

$$v_a = (v_p + v_c)\frac{f_0}{f} - v_p = (344\,m\,s^{-1} + 2.44\,m\,s^{-1})\frac{1600\,Hz}{1590\,Hz} - 344\,m\,s^{-1} = 4.62\,m\,s^{-1}$$

10. The sound intensity level decreases by $20\,dB$ at the same point of detection. What is the corresponding change of the sound intensity?

$\Delta L_I = 20\,dB$

$\Delta I = ?$

$$L_{I_1} = 10 \cdot \log\frac{I_1}{I_0} \quad ; \quad L_{I_2} = 10 \cdot \log\frac{I_2}{I_0}$$

$$\Delta L_I = L_{I_1} - L_{I_2} = 10 \cdot \log\frac{I_1}{I_0} - 10 \cdot \log\frac{I_2}{I_0} = 10\left(\log\frac{I_1}{I_0} - \log\frac{I_2}{I_0}\right) = 10\log\frac{\frac{I_1}{I_0}}{\frac{I_2}{I_0}} = 10\log\frac{I_1}{I_2}$$

$$\frac{\Delta L_I}{10} = \log\frac{I_1}{I_2} \quad ; \quad \frac{I_1}{I_2} = 10^{\frac{\Delta L_I}{10}} = 10^{\frac{20\,dB}{10}} = 100$$