

Experiment Nr. 12

DETERMINATION OF THE LINEAR EXPANSION COEFFICIENT

Theoretical part

We know that the matter expands on being heated and contracts on being cooled. The amount of expansion and contraction is greatest in gases, may be neglected in liquids and is least in solids.

The phenomena of thermal expansion in liquids and gases can be explained by the kinetic-molecular theory of the matter. According to the theory, the kinetic energy of the molecules of a matter increases with increasing temperature. As molecules acquire greater energy, they move with higher speed and collide with one another violently. The result is that the average distance between them becomes greater. Thus, the actual volume of the material expands with increasing temperature. In a limited temperature range, the volume expansion with temperature could be considered as linear.

In solids, which consist of atoms held together by elastic forces we can use a mechanical model of these forces by imagining that the atoms are connected by a set of shift springs. The interaction forces are taken to be elastic. At ordinary temperatures, the atoms vibrate about their equilibrium positions with amplitude of about 10^{-11} m and the frequency of about 10^{13} Hz. The average spacing between the atoms is about 10^{-10} m. As the temperature of solids increases, the atoms vibrate with large amplitudes and the average separation between them increases. It means that the solids expand with increasing temperature, thus, their density decrease. If the expansion of a solid is small compared with its initial dimension, then the dimension is approximately linear function of temperature.

Let us suppose that the linear dimension of a solid rod in some direction is l_0 at some temperature T_0 . The length increases by an amount of Δl for a change in temperature of ΔT . Experiments showed that the change Δl is proportional to the temperature difference ΔT as

$$\Delta l = \alpha l_0 \Delta T = \alpha l_0 \Delta t$$

where the constant of proportionality α is called the **coefficient of linear expansion** of the material, $[\alpha] = \text{K}^{-1}$. The term of the linear expansion coefficient could be introduced in case one of the solid body dimension is much larger than the other dimensions (typical for thin rods, bars or wires). The expanded length of the solid rod could be expressed as

$$l = l_0(1 + \alpha \Delta t)$$

According to the afore-mentioned formulas, the coefficient of linear expansion could be defined as follows:

$$\alpha = \frac{1}{l_0} \frac{\Delta l}{\Delta t} \approx \frac{1}{l_0} \frac{dl}{dt}$$

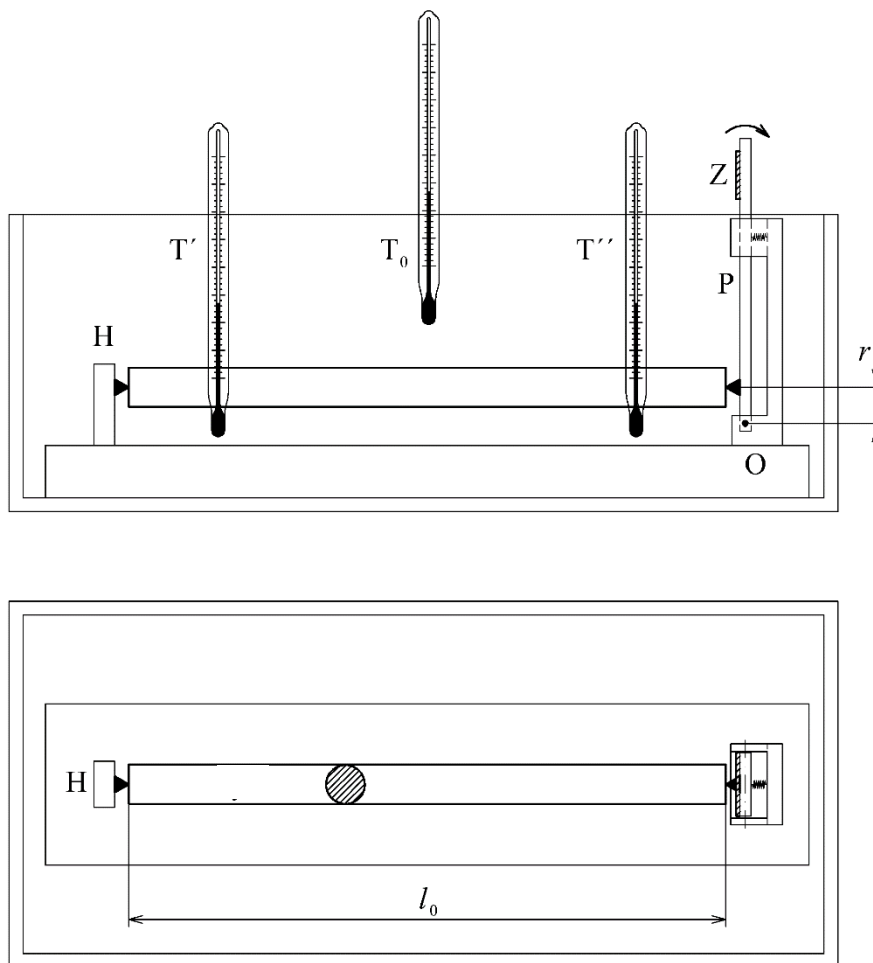
The physical unit of the coefficient α is K^{-1} . Generally, for small temperature range the coefficient of expansion is the linear function of the temperature given by an expression

$$\alpha = \alpha_1 + \alpha_2 t$$

where α_1 is of the order 10^{-5} K^{-1} and α_2 is of the order 10^{-8} K^{-1} . Typically, the temperature variation is negligible over the temperature range of about $100 \text{ }^\circ\text{C}$ and could be neglected. The coefficient of linear expansion for materials near room temperature is of order $(10^{-5} - 10^{-6}) \text{ K}^{-1}$.

Theory of the linear expansion coefficient determination

There are many methods of the linear coefficient of expansion determination. We will measure this coefficient using a device called **Edelmann's dilatometer**. The apparatus is shown in the following figure:



The Edelmann's dilatometer consists of the metal rod that is immersed into a water bath. The water medium is used due to the uniform heating of the rod. The rod is supported between two points. Because the elongation of the metal rod due to the change of the temperature is very small we have to use an appropriate displacement magnification method. Therefore, one rod end is connected to a fixed support H and the other rod end is connected to a vertical lever P with a small mirror Z . The rod expansion causes small angular motion of the mirror. A laser pointer beam reflected on the mirror generates a spot on a ruler scale placed in a far distance R . Due to rod expansion the laser spot changes its position on the scale. The change in position of the spot is proportional to the elongation of the rod. Considering the light spot position change Δn against the temperature change of Δt and the lever dimension of r , the coefficient of linear

expansion could be determined by the following formula

$$\alpha = \frac{r}{2Rl_0} \frac{\Delta n}{\Delta t} = \frac{r}{2Rl_0} K$$

The coefficient K should be determined as the slope coefficient of the linear function given by

$$n_i = n_0 + Kt_i$$

where n_0 is the light spot position at 0 °C.

Measurement objectives

1. Plot the function $n_i = n_0 + Kt_i$ for at least 8 pairs of the values n_i, t_i in the temperature range from room temperature to about 70 °C.
2. Determine the coefficient of linear expansion for one selected rod sample and evaluate the measurement uncertainty. Compare the obtained result to the material property given by the tabular data.

Important constants

The lever dimension is $r = (15,00 \pm 0,30)$ mm

The length of the rod is $l_0 = (500,0 \pm 1,0)$ mm

Other notes

The measuring instrument base is made of granite. Because the granite base is immersed in the water as well, we have to count with the expansion coefficient of granite in our results. The expansion coefficient of granite is $\alpha_g = 8 \cdot 10^{-6} \text{ K}^{-1}$. The base expansion lowers the resulting value of the metal rod expansion coefficient. Thus, we have to add the value of α_g to our metal rod expansion coefficient to obtain relevant metal expansion coefficient value.

Uncertainty calculation notes

The combined uncertainty consist of both Type A and Type B uncertainty. The relative Type A uncertainty can be determined by

$$u_{r\alpha A} = u_{rKA},$$

where u_{rKA} is the slope coefficient uncertainty given by the regression calculation results.

The relative Type B uncertainty can be calculated from the Type B sub uncertainties of the r , R , and l_0 values as follows:

$$u_{r\alpha B} = \sqrt{u_{rrB}^2 + u_{rRB}^2 + u_{rl_0B}^2}$$

The combined standard uncertainty of the linear expansion coefficient is

$$u_{r\alpha} = \sqrt{u_{r\alpha A}^2 + u_{r\alpha B}^2}$$