Experiment Nr. 13

DETERMINATION OF THE VISCOSITY OF GLYCERINE USING STOKES'S LAW

Theoretical part

The viscosity of liquids

Viscosity is a measure of the resistance of real liquids to deform under shear stress. The shear stress can be defined as the proportion of the shear force and the area which the force acts on. This resistance occurs in all the liquids, however, not in the ideal one. The ideal liquid is defined as incompressible and perfect fluid in its motion, i.e. without resistance. However, (perfect) ideal liquids do not actually exist, and different (real) liquids have different properties of fluidity. For example, glycerine flows more slowly than water under the same conditions. The reason for an imperfect fluidity is a friction between the layers inside the liquid, acting against their relative movement. This interaction is similar to the friction between the two surfaces of solid objects, while instead of the surface it takes place in bulk in a case of liquids. **Viscosity** is a physical property describing this inner friction. Bigger the liquid's viscosity is, more the flow speed of this liquid is reduced and/or more the movement of an object is slowed down in this liquid.

Figure 1 shows a schematic image describing the mechanism of viscosity. The liquid is enclosed in between two parallel plates, which are apart from each other by the distance *y*. The bottom plate is fixed, while the top one moves with a speed ***v*** horizontally. On both solid plate - liquid interfaces, there is a thin layer of liquid attached firmly to the plate by adhesive forces. Thus, the very top layer of liquid moves together with a moving plate, while the very bottom layer does not move at all. The inner friction of liquid then causes the division of bulk liquid into several layers moving with a speed that changes linearly from zero to the maximum speed *v* approximately.

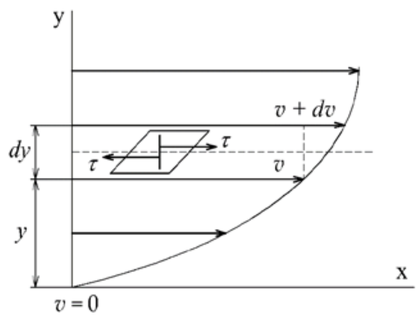

Fig.1 The division of bulk liquid into several layers caused by the movement of the top plate.

The continuous movement of the top plate must be enforced by a tangential force *F*, which cancels the inner friction inside the liquid. Assuming the low differential speed dv and small thickness dy (so called velocity gradient), it can be approximated by a formula

$$F = \eta S \frac{dv}{dy} \qquad (1)$$

where S is the area of the moving plate. The linear constant η is a coefficient of dynamic viscosity, called also **dynamic viscosity**. According to equation (1) it is expressed in units Pa.s. The ratio of the liquid's dynamic viscosity η to its density ρ defines the **kinematic viscosity:**

$$\nu = \frac{\eta}{\rho} \qquad (2)$$

that is expressed in units $m^2.s^{-1}$. The value of viscosity varied from liquid to liquid. In most of the cases, the viscosity is inversely proportional to the temperature (decreasing exponential dependence typically). The viscosity of the gases is much lower that that of the shows a increasing value with increasing temperature, typically.
Most of the liquids, such as water, are described by the equation (1) and are known as Newtonian liquids (Fig. 2). Note, that the formula (1) assumes a linear equation passing through the origin. Non-Newtonian liquids, on the other hand, exhibit more complicated relationship between shear stress and velocity gradient than simple linear. Also it should be noted, that according to the definition of the ideal liquid, its viscosity is equal to zero. The curves describing the dependence of the shear stress against the velocity gradient are commonly known as rheograms (Fig. 2).

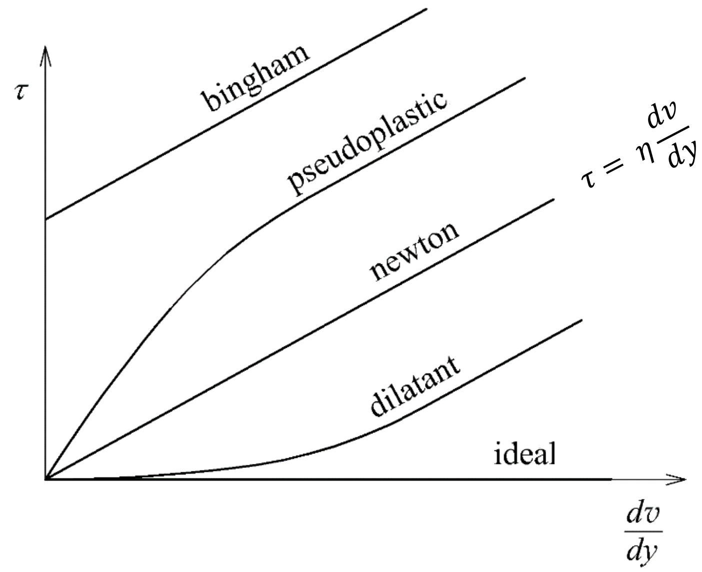

Fig.2 Rheograms for typical liquids.

The falling ball viscometers

The falling-body viscometers are based on the fact that the viscosity of liquid modifies the velocity of ball falling in this liquid. For these viscometers, the laminar flow is assumed. When the ball of volume *V*, radius *r* and mass *m* is descending in the liquid, it is affected by the forces shown in Fig. 3.

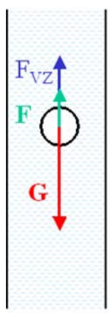

Fig. 3. The forces experienced by the ball falling in the liquid.

Downwards acting force is given by the weight of ball:

$$G = mg = V\rho g = \frac{4}{3}\pi r^3 \rho g \qquad (3)$$

where r is the ball radius, ρ is the density of ball and g is gravitational acceleration. In an opposite direction, the ball is pulled up by the buoyant force F_{VZ} according to the Archimedes principle. This force is equal to the weight of liquid displaced by the ball

$$F_{VZ} = V\rho g = \frac{4}{3}\pi r^3 \rho_k g \quad (4)$$

where ρ_k is the density of liquid. Finally, real liquid with dynamic viscosity η generates a resistance against the movement of ball. This resistive force F can be derived from the Stokes' law

$$F = 6\pi\eta r v \quad (5)$$

where v is the velocity of ball. While the two previously mentioned forces (G and F_{VZ}) are static and do not depend on the speed v, the resistance F increases with the speed. Thus, the velocity of the falling ball will be increasing only until the net force is zero

$$G - F_{VZ} - F = 0 \quad (6)$$

The combination of equations (3-5) with (6) yields

$$v = \frac{2}{9} r^2 g \frac{\rho - \rho_k}{\eta} \quad (7)$$

According to the equation (7), the viscosity of liquid can be determined from the velocity of a ball descending in this liquid. Stokes's law (5) and formula (7) are valid in the case, when a body moves uniformly, without the rotation or turbulence, in a homogeneous liquid which has no bounds. The Stokes's law validity can be verified by the Reynolds criteria of the laminar flow. The Reynolds number is

$$\mathrm{Re} = \frac{vd}{\nu} \quad (8)$$

In the laminar flow regime (Re < 0.5), the resistivity coefficient C can be obtained from the Reynolds number by

$$\mathrm{C} = \frac{24}{\mathrm{Re}} = \frac{8}{3} r g \frac{\rho - \rho_K}{v^2 \rho_K} \quad (9)$$

Measurement objectives

1. Determine the average speed of the glass balls descending in the glycerine.
2. Check the validity of the Stokes's law for your case.
3. Determine the coefficient of dynamic viscosity and evaluate the measurement uncertainty.
4. Calculate the kinematic viscosity and evaluate the measurement uncertainty.

Measurement procedure

On the measuring cylinder, there are two metal rings for free fall path marking. Shift these rings downwards and set a path of about 10 cm. Determine the falling velocity for one measurement and check the validity of the Stokes's law - calculate the resistivity coefficient using formula (9) and validate the criteria Re < 0.5.

If the laminar flow is confirmed, move the rings to a path of about 30 cm and carry out 10 measurements of the falling velocity. Calculate the average velocity and its uncertainty (see below).

Finally, evaluate the coefficient of dynamic viscosity using the formula (7) and kinematic viscosity using formula (2).

Important constants

Glass ball diameter is d = (3,450 ± 0,020).10^{-3} m

Glass density ρ = (2580 ± 10) kg.m^{-3}

Uncertainty calculation notes

The dynamic viscosity measurement should count both Type A and Type B uncertainty. In the case of the Type A uncertainty, there is only one source - the repeated measurement of time, thus

$$u_{r\eta A} = u_{r\tau A}$$

The Type B uncertainty has more sources. The sum of quadrature of all sub uncertainties lead to a formula

$$u_{r\eta B} = \sqrt{\frac{u_{\rho B}^2 + u_{\rho_k B}^2}{\left(\rho - \rho_k\right)^2} + 4u_{rdB}^2 + u_{rsB}^2}\ ,$$

where $u_{rdB} = u_{rrB}$ is the relative uncertainty of the ball diameter (equals to the relative uncertainty of the ball radius) and u_{rsB} is the uncertainty of the free fall path measurement (determined by the tape measure). The standard combined uncertainty of the dynamic viscosity is

$$u_\eta = \sqrt{u_{\eta A}^2 + u_{\eta B}^2}\ \text{, event. }\ u_{r\eta} = \sqrt{u_{r\eta A}^2 + u_{r\eta B}^2}$$

The relative uncertainty of the kinematic viscosity is given by the formula

$$u_{r\nu} = \sqrt{u_{r\eta}^2 + u_{r\rho}^2}\ ,$$

where $u_{r\rho}$ is the relative uncertainty of the liquid density determination.