# Experiment Nr. 25

## DETERMINATION OF SOUND INTENSITY USING RAYLEIGH PLATE

### Theoretical part

Acoustic waves (sound) can propagate within a material substance called a medium (solid, liquid, gaseous state and plasma). A sound wave is a travelling longitudinal elastic wave, thus, it causes a periodic compression and dilution of the medium. Near a point source, the wave-fronts are spherical and are spreading out in three dimensions (spherical waves). As the wave-fronts move outward and their radii become larger, their curvatures decrease. Far from the source they can be assumed to be planar.
The human ear is sensitive to sound waves in the frequency range from about 16 Hz to 20 kHz. The sound waves with frequencies higher than 20 kHz are called as ultrasound. The sound waves with frequencies lower than 16 Hz are called as infrasound.
The simplest sound waves are sinusoidal waves with definite frequency, amplitude and wavelength. A sinusoidal sound wave in an elastic medium is described by the following wave function

$$u = u_0 \sin \omega \left( t - \frac{x}{c} \right)$$

where $u_0$ is the wave amplitude, $\omega$ is the angular velocity, $t$ is time and $c$ is the phase velocity of the wave in the selected medium. The wavelength $\lambda$ and the wave frequency $f$ define the phase velocity by following formula

$$\lambda = \frac{c}{f}$$

Sound waves can also be described in terms of the air pressure variation. The pressure fluctuates above and below the atmospheric pressure. Its variation in time is sinusoidal and has the same angular frequency $\omega$ as that of the motion of air particles. The sound pressure variations are given by

$$p = p_0 \cos \omega (t - \frac{x}{c}) \text{ , } [p] = \text{Pa}$$

Thus the relation between the pressure and the sound wave function is

$$p = \rho v c$$

where $\rho$ is the medium density and $v$ is the sound particle velocity (it's the velocity of the wave oscillations, not the phase velocity !). The pressure amplitude for the lowest detectable sound intensity at the frequency 1 kHz is about $2.8 \cdot 10^{-5}$ Pa. The root-mean-square (RMS or effective) value of the pressure is

$$p_{ef} = \frac{p_0}{\sqrt{2}}$$

The maximum pressure amplitude $p_0$ of a sound wave that the human ear can tolerate is about 28 Pa (much less than the normal atmospheric pressure of about $10^5$ Pa).
The intensity $I$ of a travelling wave is defined as the time average rate at which energy is transported by the wave, per unit area, across a surface perpendicular to the direction of propagation. Briefly, the intensity is the average power transported per unit area that is perpendicular to the direction of the wave propagation. The intensity of as sound wave can be expressed as

$$I = \frac{p_{ef}^2}{\rho c}, \ [I] = \mathrm{W} \cdot \mathrm{m}^{-2}$$

where the speed of sound $c$ is temperature sensitive according to the relation (temperature $t$ is in °C and speed in $\mathrm{m.s^{-1}}$)

$$c = 344.3 + 0.62\,(t - 20)$$

Because of extremely large range of intensities over which the human ear is sensitive (up to 12 orders of magnitude), a logarithmic rather than arithmetic intensity scale is convenient. The intensity level $L_I$ of a sound wave is defined by

$$L_I = 10 \log \frac{I}{I_0}, \ [L_I] = \mathrm{dB}$$

where $I_0$ is the threshold of hearing intensity: $I_0 = 1.10^{-12}$ $\mathrm{W.m^{-2}}$.
Within the range of audibility, the sensitivity of the ear varies with frequency. The experimental finding of human perception of sound is graphically expressed in the following figure as a set of curves of certain level of intensities in the $L_I$ vs. frequency $f$ diagram (nomogram, see Fig. 1). The level of sound intensity is then expressed as the loudness level $L_H$ with physical unit ph ("phon"). Note that for the sound frequency of 1 kHz, the loudness level equals numerically to the value of the intensity level.

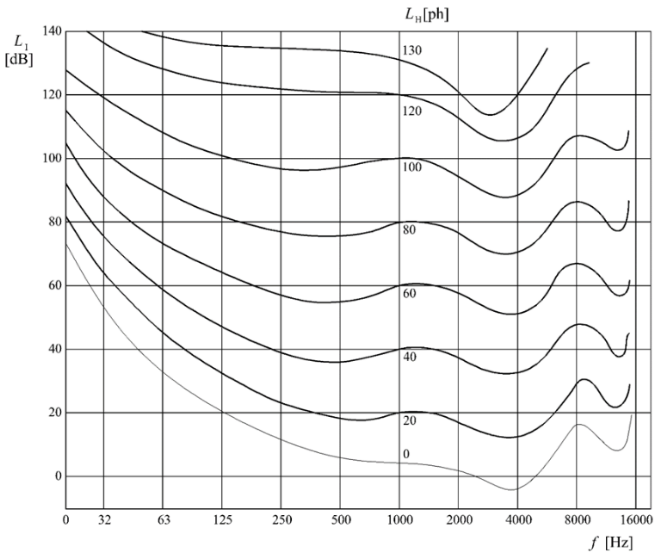


**Fig. 1** Nomogram for determination of the loudness level

## Theory of the measurement principle

The method of determining the sound intensity by the Rayleigh plate is based of an angular displacement measurement of a small circular plate made of mica. The plate is suspended on a thin wire (see Fig. 2).

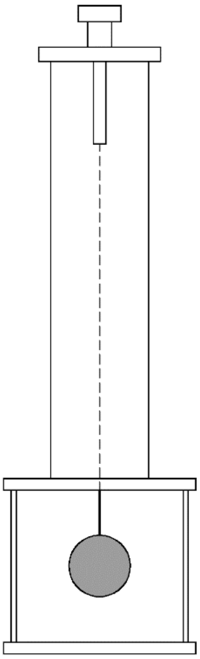

**Fig. 2** The Rayleigh plate instrument

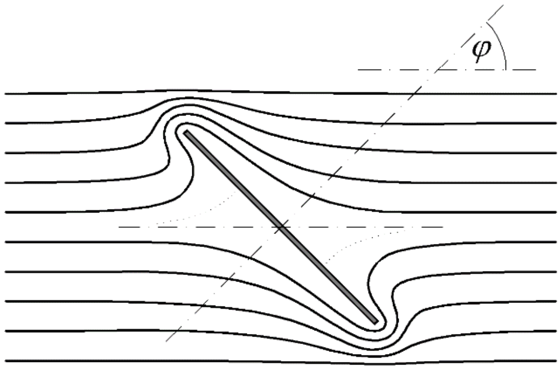


**Fig. 3** Description of the circular plate position

A small mirror is angularly fixed to the plate, so that the angular displacement could be observed by the laser pointer beam reflected on the mirror. The sound waves generated by a sound speaker are directed to toward the plate.
A field of sound wave streamlines around the plate is shown in Fig. 3. The extreme values of air pressure are at the locations with the lowest streamlines density (the strongest effect can be obtained for the 45° declination of the plate). Due to this, a torque is acting on the plate. When the plate is exposed to the sound flow, it starts to rotate. The equilibrium angular displacement $\varphi$ of the plate is due to the equality of the external torque (generated by the sound flow) and the restoring torque of the wire. Considering this equilibrium and the definition of sound wave intensity we can get following equation

$$I = \frac{3}{4}\frac{c}{r^3} J \left(\frac{\pi}{\tau}\right)^2 \frac{\Delta n}{2R}$$

where $r$ is the plate radius, $J$ is the moment of inertia of the rotating system against the vertical axis, $\Delta n = n - n_0$ is the change of the equilibrium angular position determined as the laser beam position change on the far-distant scale when the sound source is off ($n_0$) and on ($n$), $R$ is the mirror-scale distance (total path of the reflected laser beam), $c$ is the phase speed of the sound, $\tau$ swing time of the undamped oscillating system (sound is off), that could be determined from the real damped system swing time $\tau_t$ using following formula

$$\tau = \frac{\tau_t}{\sqrt{1+\frac{\vartheta}{2\pi}}},$$

where $\vartheta$ is logarithmic decrement showing the damping ratio of the plate oscillating system. The logarithmic decrement can be obtained from

$$\vartheta = \ln\frac{a_1 - n_0}{a_3 - n_0},$$

where $a_1$, $a_3$ are the outer swing positions on one side of the oscillating laser beam spot, see Fig. 4. Note that the $n_0$ position in the logarithmic decrement correspond to the measurement when the sound is off, thus, the $a_1$, $a_3$ positions should be taken from the data when the sound is off as well.

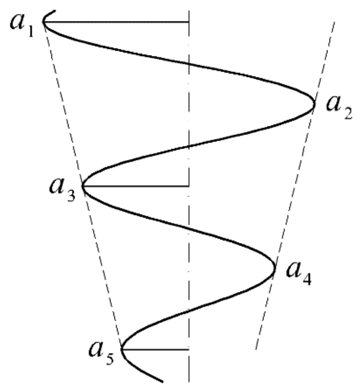


**Fig. 4** The systematic plot of the outer swing positions description

According to the description of the outer swings position, the equilibrium position of the oscillating laser beam spot can be calculated from the 3-swings approach by

$$n_0 = \frac{1}{2}\left[a_2 + \frac{1}{2}(a_1 + a_3)\right]$$

where $a_1$, $a_2$ and $a_3$ are three consecutive outer swings of the laser spot. Use the similar formula for determination of the equilibrium position $n$ when the sound is on as well.

## Measurement objectives

1. Determine the equilibrium position $n$ of the Rayleigh plate oscillating system when the sound wave source is on.
2. Determine the swing time $\tau$ of the damped plate system.
3. Calculate the sound wave intensity, acoustic pressure RMS value, and the sound intensity level.
4. Determine the loudness level using the nomogram (Fig. 1).

## Measurement procedure

First, determine the equilibrium positions $n$ and $n_0$ using the 3-swings approach (see above). Using the stop-watch carry out a measurement of the laser beam spot swing time when the sound wave source is off (use an appropriate method for lowering the measurement uncertainty and consider the time uncertainty determination). Last, determine the mirror-scale distance $R$.
Calculate the logarithmic decrement and the undamped system swing time. Using these afore-mentioned results, speed of sound determined for the lab temperature and other constants (see below), calculate the sound wave intensity $I$ and corresponding intensity level $L_I$.
Using the nomogram (Fig. 1) determine the loudness level $L_H$ (the sound wave frequency should be 1 kHz).

## Important constants

Moment of inertia of the Rayleigh plate is $J = (1{,}325 \pm 0{,}013)\cdot 10^{-7}$ kg.m$^2$.
The Rayleigh plate radius is $r = (22{,}0 \pm 0{,}20)$ mm.
The density of the ambient air is $\rho = 1.2$ kg.m$^{-3}$.

## Uncertainty calculation notes

### *Sound wave intensity I*

The combined uncertainty of the sound wave intensity $I$ consist of both Type A and Type B uncertainty. The relative Type A uncertainty has only one source (swing time) and can be determined by

$$u_{rIA} = u_{r\tau A}$$

where $u_{r\tau A}$ is the relative uncertainty of the swing time measurement determined using either the standard deviation of the swing time data or the slope coefficient uncertainty given by the regression calculation results. The calculation should be carried out according to the swing time measurement method.
The relative Type B uncertainty of intensity $I$ can be calculated from the Type B sub uncertainties of the values as follows:

$$u_{rIB} = \sqrt{9u_{rrB}^2 + u_{rRB}^2 + u_{rJB}^2 + \frac{u_{nB}^2 + u_{n_0B}^2}{\left(n - n_0\right)^2}}$$

### *Acoustic pressure $p_{ef}$*

The acoustic pressure uncertainty can be determined using the theory of summation of quadrature law. The uncertainties of $c$ and $\rho$ can be neglected. We can conclude that only Type B uncertainty has to be considered, thus

$$u_{rp_{ef}B} = \frac{1}{2}u_{rIB}.$$