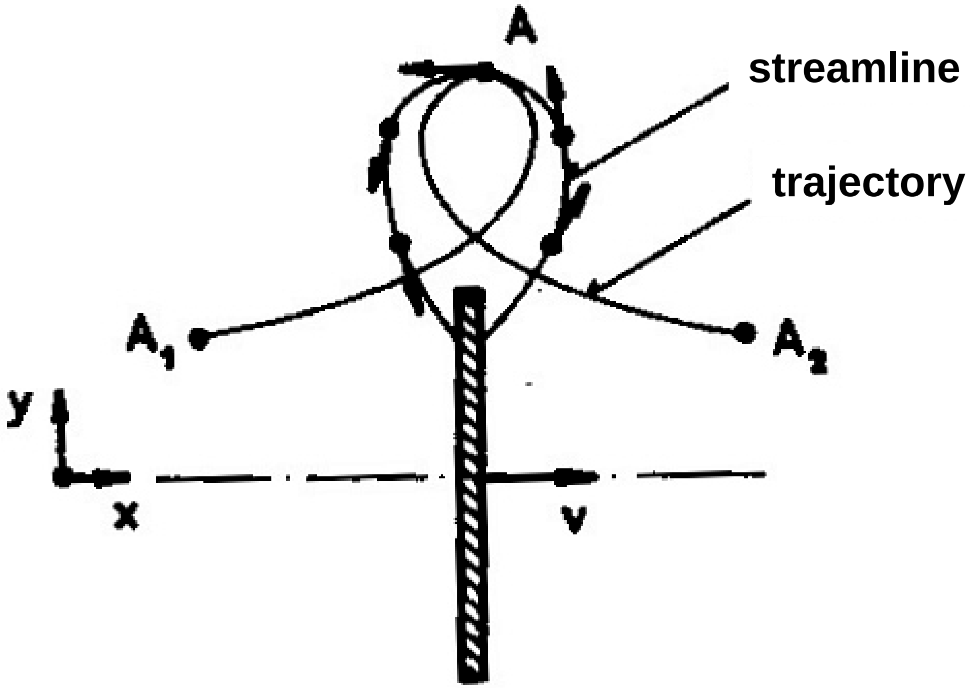
Fluid Mechanics

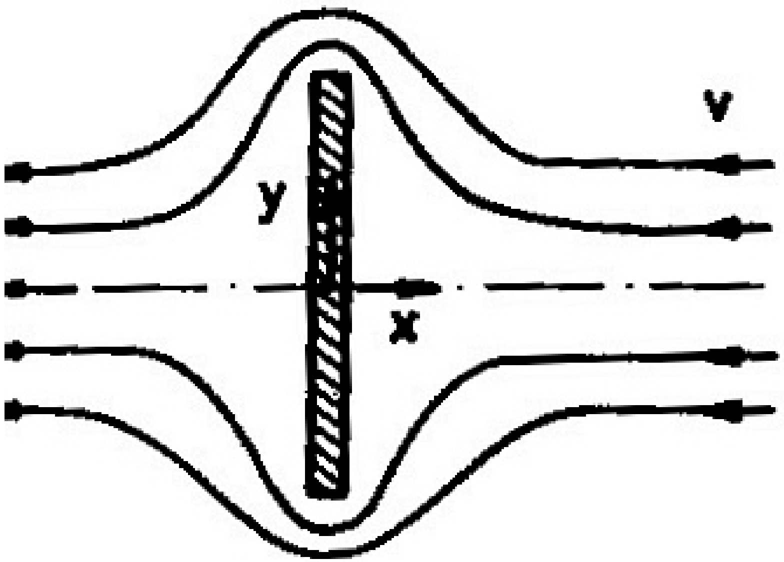
Fluid mechanics problems

fluids – liquids and gases
compressible, incompressible

flow – laminar
turbulent

ideal (perfect) fluid





flow velocity description – Lagrangian a Eulerian

Streamlines

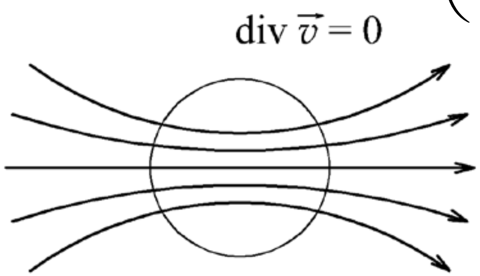
sources and sinks ⟶ velocity divergence

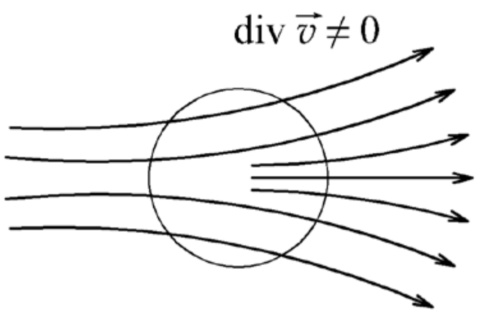
$$\operatorname{div}\vec{v} = \vec{\nabla}\cdot\vec{v}$$

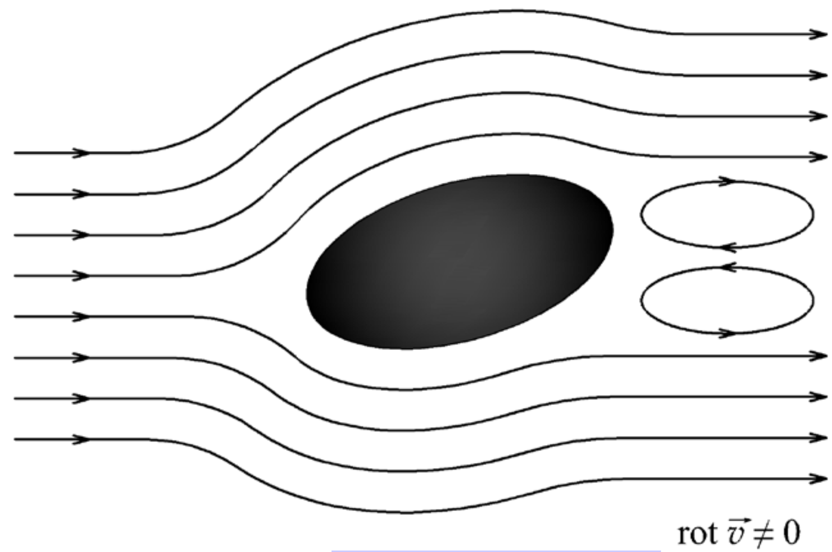
$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\operatorname{div}\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\operatorname{rot}\vec{v} = \vec{\nabla}\times\vec{v}$$







$\operatorname{div}\vec{v} \neq 0$ **source / sink flow**

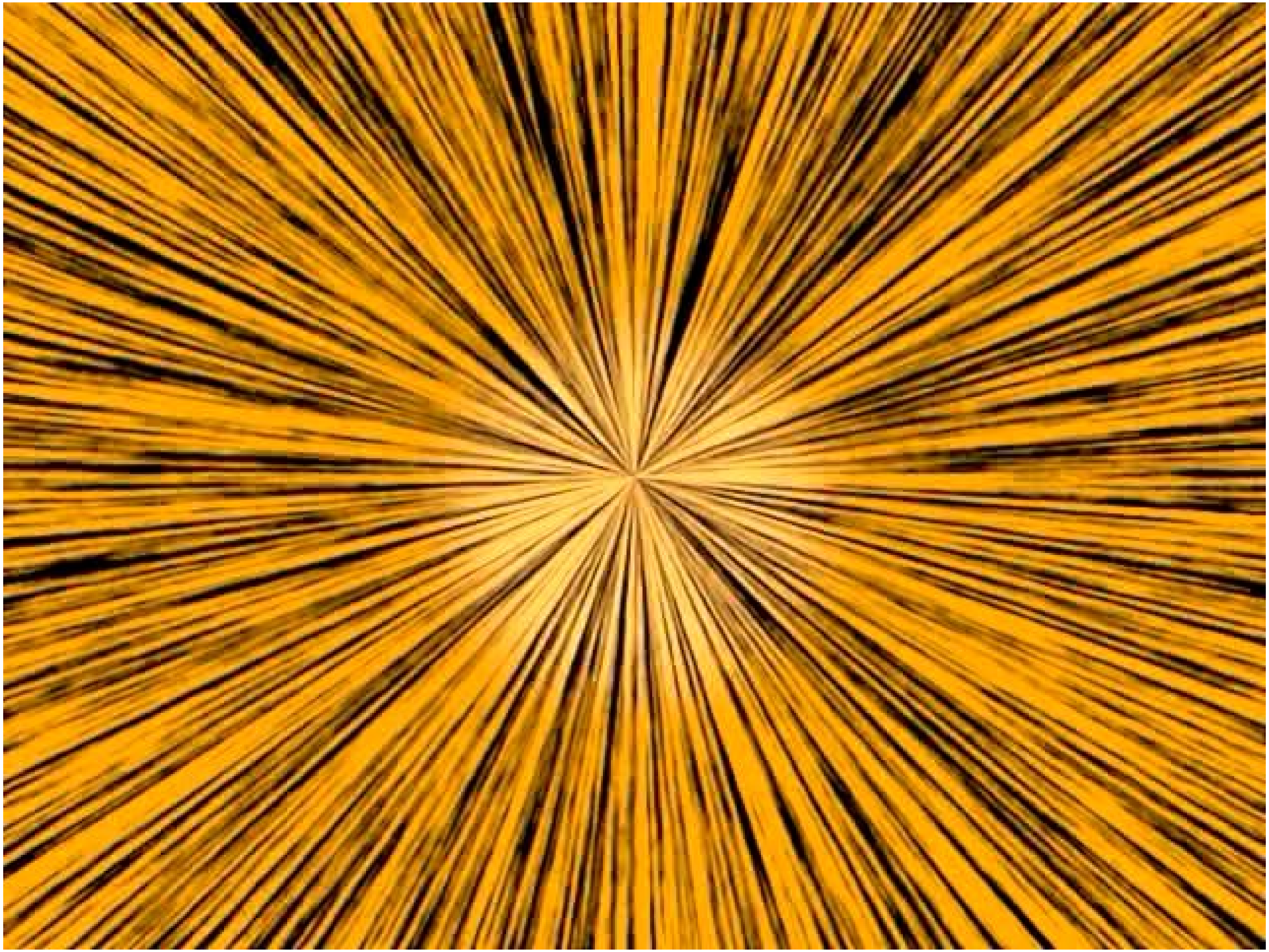
$\operatorname{rot}\vec{v} \neq 0$ **vorticity**

translation + rotation

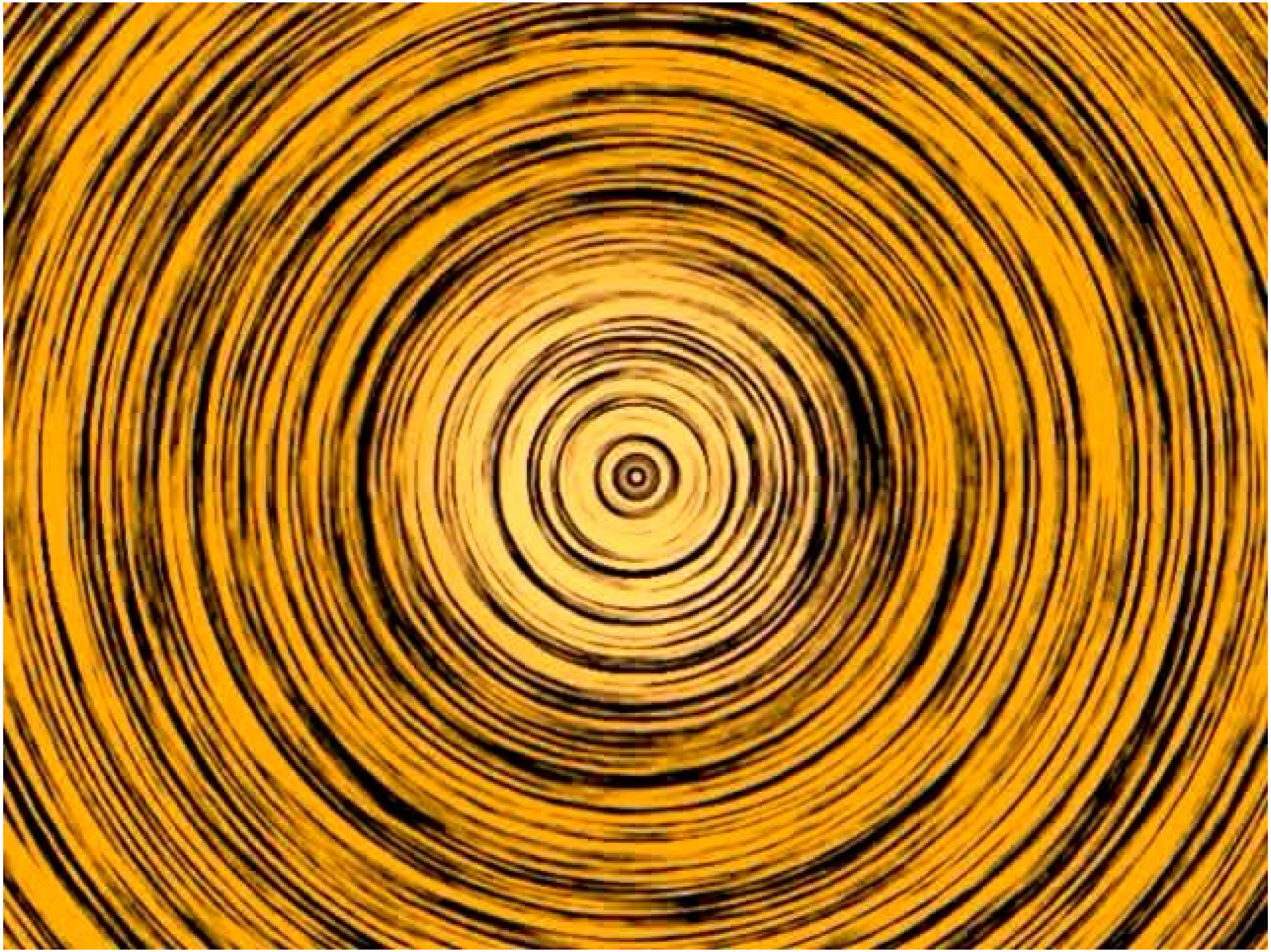
$$\vec{v} = \vec{v}_T + (\vec{\omega}\times\vec{r})$$

$$\operatorname{rot}\vec{v} = \operatorname{rot}\vec{v}_T + \operatorname{rot}(\vec{\omega}\times\vec{r})$$

$$\operatorname{rot}\vec{v} = 2\,\vec{\omega}$$



Fluid flow with a source

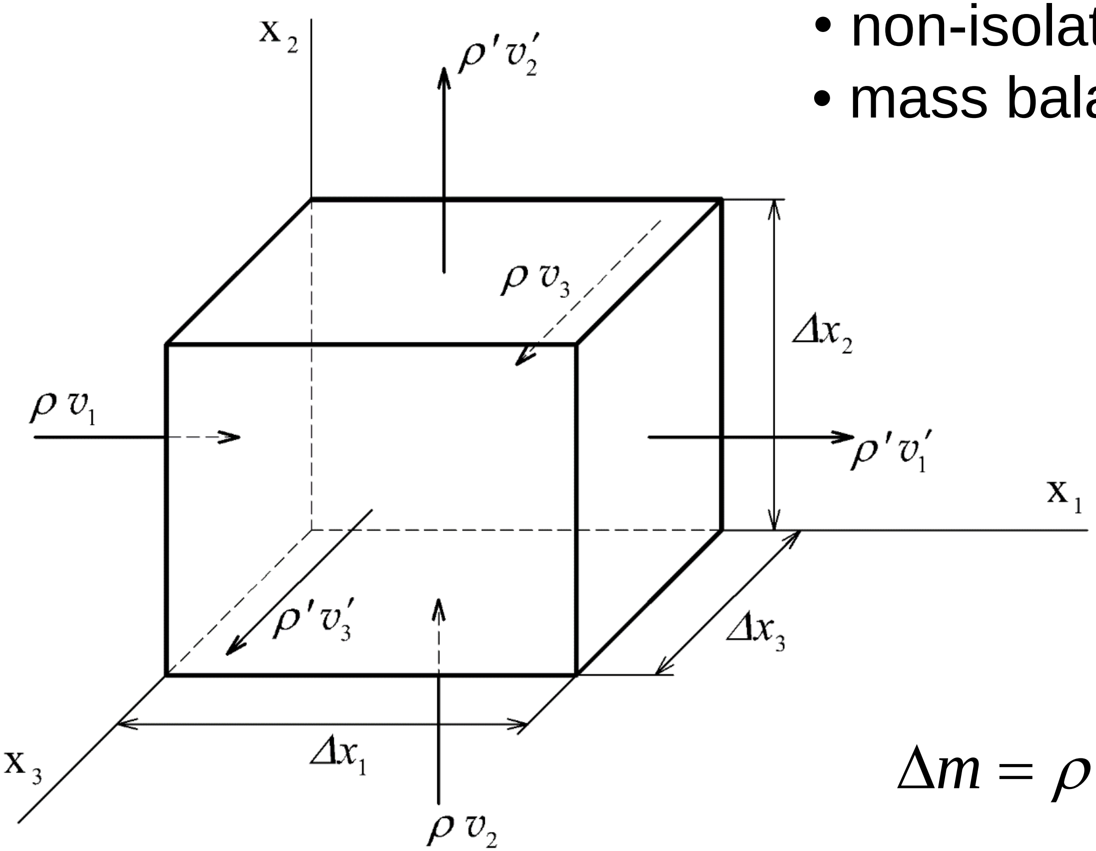


Fluid flow with a vortex

Fluid flow with source and vortex

Mass Balance Equation

- non-isolated system
- mass balance equation for ΔV



average mass flux

$$\vec{j}_m = \rho\vec{v}$$

$$\Delta m = \rho\,\Delta V$$

$$-\Delta m = \left[(\rho' v_1' - \rho v_1)\Delta S_1 + (\rho' v_2' - \rho v_2)\Delta S_2 + (\rho' v_3' - \rho v_3)\Delta S_3\right]\Delta t$$

conversion $\Delta m \,/\, (\Delta V \Delta t)$

$$-\frac{\Delta m}{\Delta V}\frac{1}{\Delta t} = \frac{\Delta(\rho v_1)}{\Delta x_1} + \ \ldots$$

$$\mathrm{div}\rho\vec{v} = \sum_{k=1}^{3} \frac{\partial(\rho v)_k}{\partial x_k} = \frac{\partial(\rho v_1)}{\partial x_1} + \frac{\partial(\rho v_2)}{\partial x_2} + \frac{\partial(\rho v_3)}{\partial x_3}$$

$$-\frac{\partial \rho}{\partial t} = \mathrm{div}\,\rho\vec{v}$$

Continuity Equation

$$\iiint_V \mathrm{div}\left(\rho\vec{v}\right) dV = -\iiint_V \frac{\partial \rho}{\partial t} dV$$

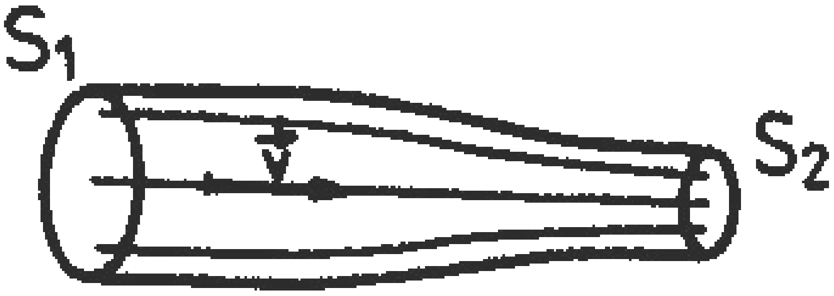
$$\oiint_S \rho\vec{v}\cdot d\vec{S} = \iiint_V \mathrm{div}\left(\rho\vec{v}\right) dV$$

$$\iiint_{\Delta V} \frac{\partial \rho}{\partial t} dV + \oiint_{\Delta S} \rho\vec{v}\cdot d\vec{S} = 0$$

An increase of mass *dm* within a volume *dV* equals to the difference of the inward mass flow and the outward mass flow.

Stationary (Steady) Flow

$$\frac{\partial\rho}{\partial t} = 0 \Rightarrow \operatorname{div}\rho\vec{v} = 0$$



$$\Delta m_1 = \rho_1 S_1 v_1 \,\Delta t$$

$$\rho_1 v_1 S_1 = \rho_2 v_2 S_2$$

$$\Delta m_2 = \rho_2 S_2 v_2 \,\Delta t$$

incompressible fluid

$$\frac{\partial\rho}{\partial t} = 0 \qquad \operatorname{grad}\rho = 0$$

$$v_1 S_1 = v_2 S_2$$

$$\operatorname{div}\vec{v} = 0$$

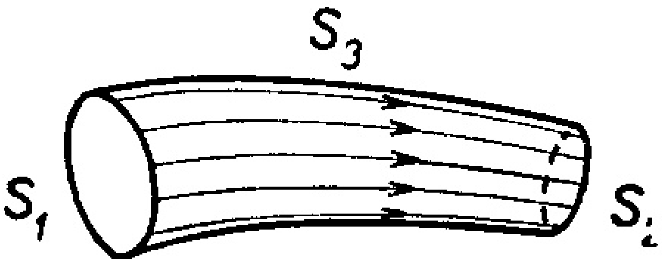
$$\mathrm{rot}\,\vec{v} = 2\vec{\omega} = \vec{\Omega}$$

vorticity

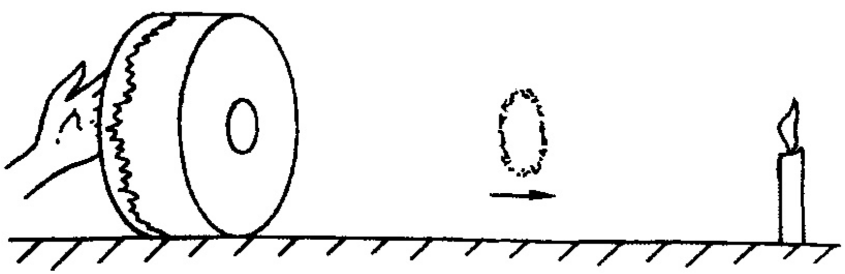
field of vortices $\mathrm{div}\,\vec{\Omega} = 0$ solenoidal field

streamlines = vortex filament

stream tube = vortex tube



vortex intensity $\mu = \iint_S \vec{\Omega} \cdot d\vec{S}$



Balance Equations of Linear Momentum and Energy

$$\Delta m\vec{a} = \Delta\vec{F}_O + \Delta\vec{P}_t + \Delta\vec{P}_V$$

$$\lim_{\Delta V\to 0}\frac{\Delta m}{\Delta V} = \rho \qquad \lim_{\Delta V\to 0}\frac{\Delta\vec{F}_o}{\Delta V} = \rho\vec{K}$$

$$\lim_{\Delta V\to 0}\frac{\Delta\vec{P}_t}{\Delta V} = -\mathrm{grad}\; p \qquad \lim_{\Delta V\to 0}\frac{\Delta\vec{P}_V}{\Delta V} = \vec{f}_V$$

$$\rho\vec{a} = \rho\vec{K} - \mathrm{grad}\; p + \vec{f}_V$$

motion equation for a viscous fluid

$$\rho\vec{a} = \rho\vec{K} - \mathrm{grad}\; p$$

Euler equation (non-viscous fluid)

$$\rho\vec{a} = -\rho\,\mathrm{grad}\,U - \mathrm{grad}\; p$$

$$\vec{a}\cdot d\vec{r} = -dU - \frac{1}{\rho}dp$$

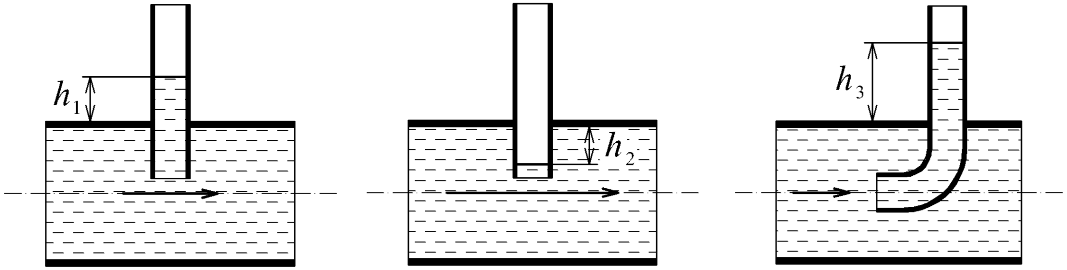
$$\vec{a}\cdot d\vec{r} = \sum_{i=1}^{3}\frac{dv_i}{dt}dx_i = \sum_{i=1}^{3}dv_i v_i = d(\frac{1}{2}v^2)$$

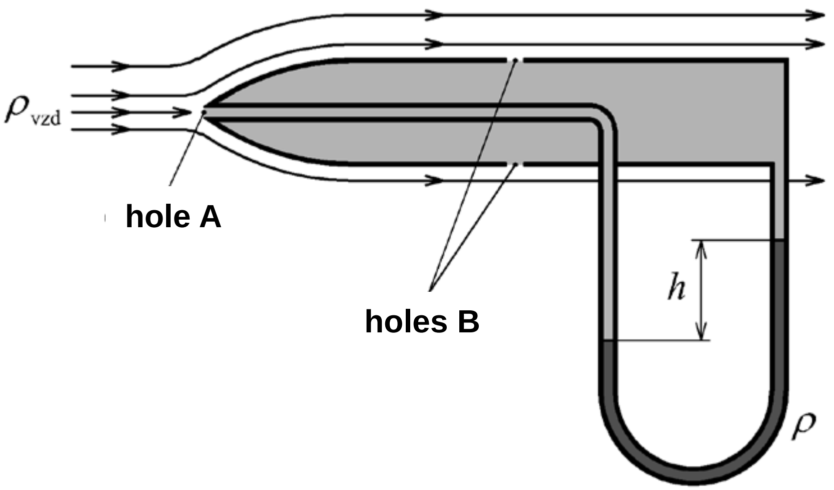
$$d(\frac{1}{2}v^2) + dU + \frac{1}{\rho}dp = 0$$

Bernoulli equation

$$\frac{1}{2}\rho v^2 + \rho U + p = \mathrm{konst}$$

Pitot Tube



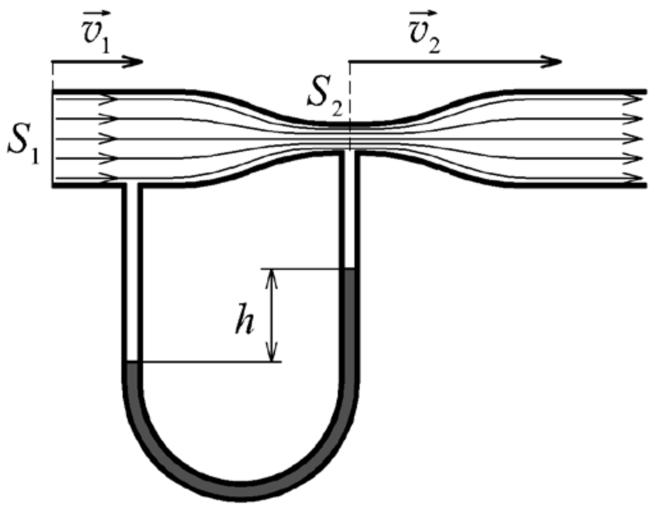


Pitot-static (Prandtl) tube

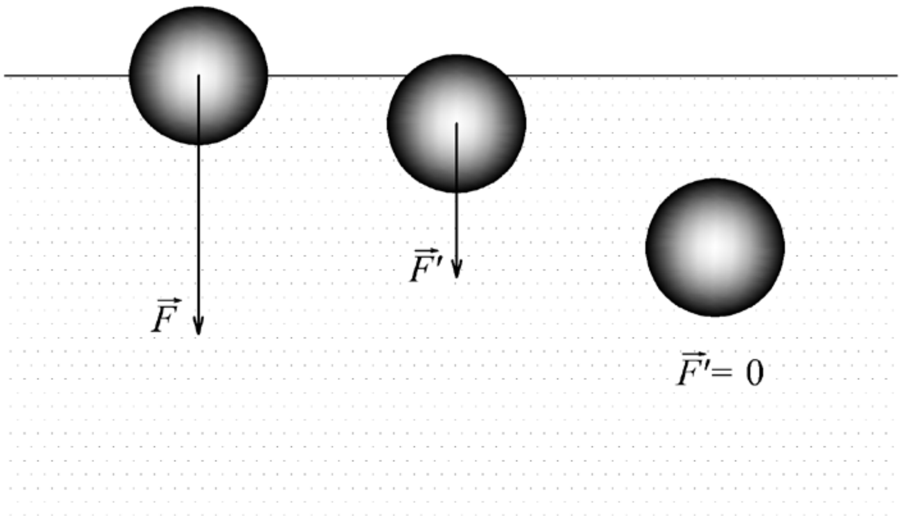
$$v = \sqrt{\frac{2\rho g h}{\rho_{\text{vzd}}}}$$

Venturi tube (flow meter)

$$v = \sqrt{\frac{2s^2 \Delta p}{\rho\left(S^2 - s^2\right)}} \qquad \Delta p = \rho_{\text{n}} g h$$

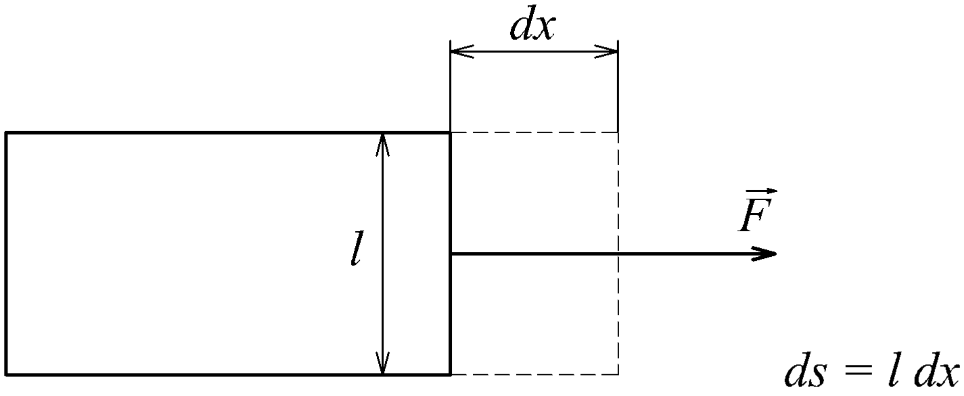


Surface Tension in Liquids

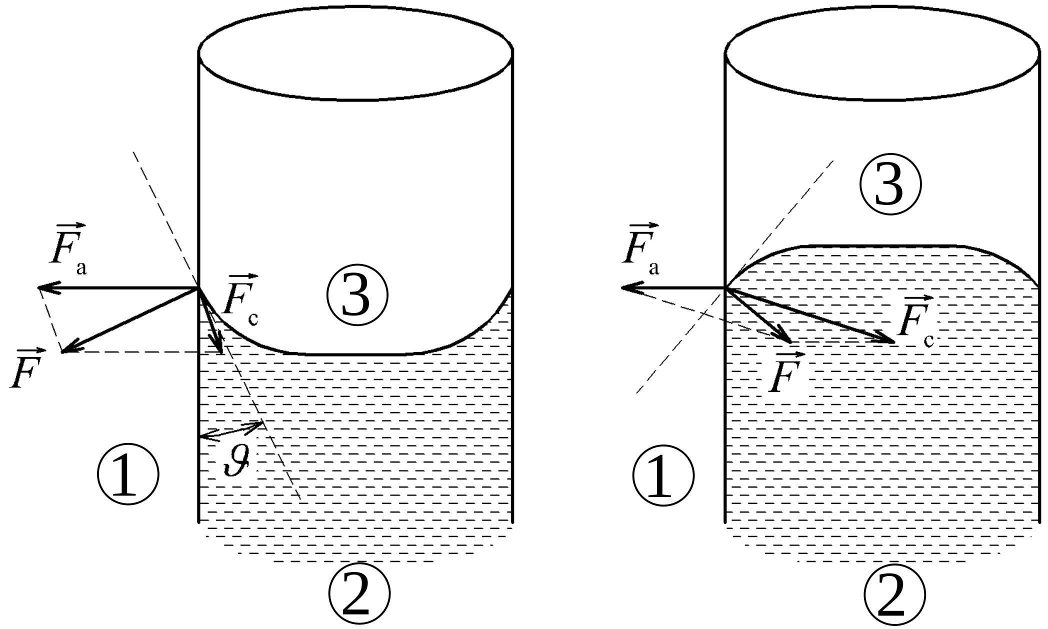


Surface tension equals to the work *dA* of the molecular interaction forces needed for a surface area *dS* generation.

$$\sigma = \frac{\partial A}{\partial S}$$



$$\sigma = \frac{F}{l}$$

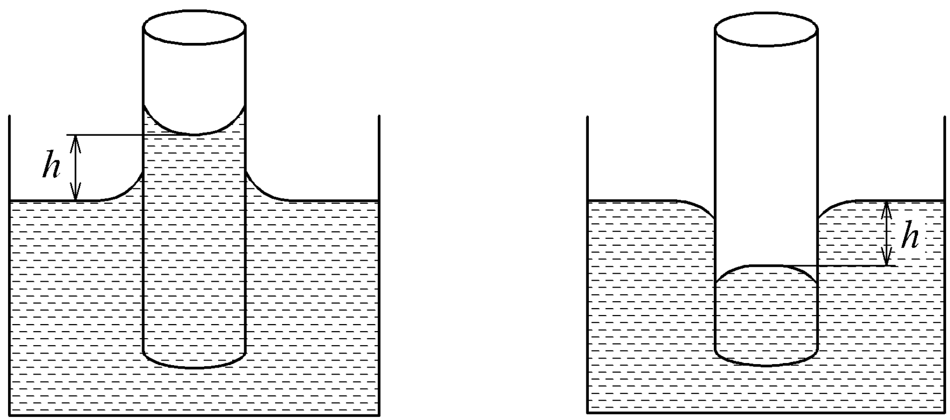


wetting

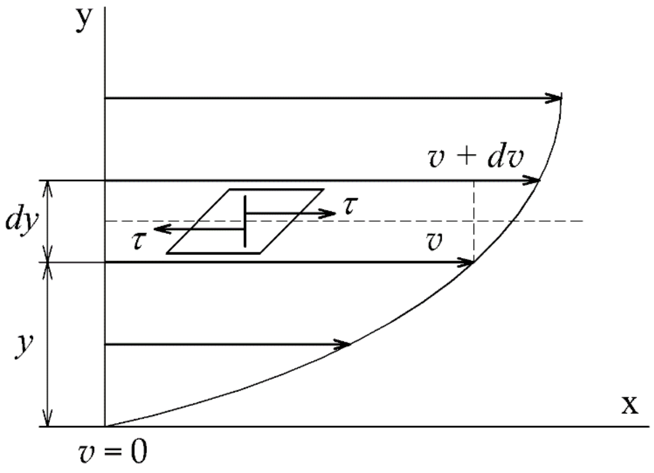
$\sigma_{13} > \sigma_{12}$

non-wetting

$\sigma_{13} < \sigma_{12}$

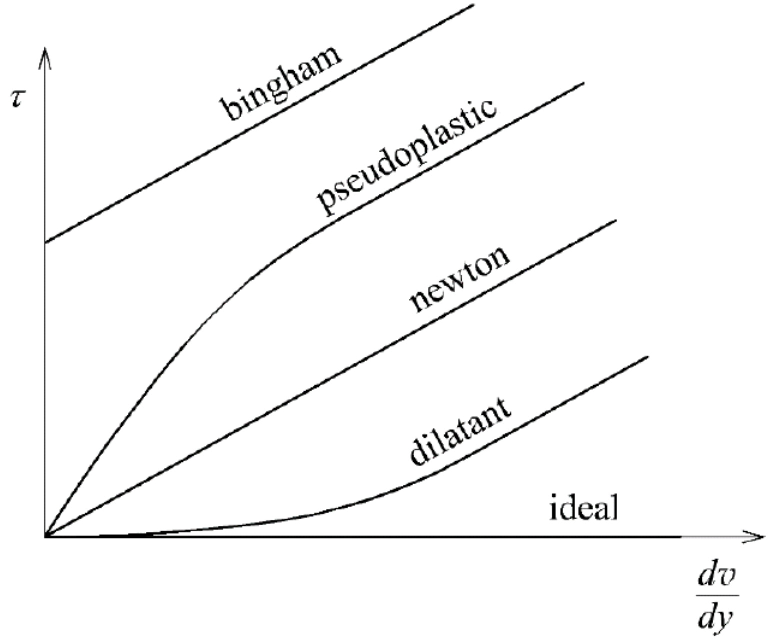


Viscosity



$$\tau = \eta \frac{dv}{dy}$$ Newton's law of viscosity

η dynamic viscosity coefficient



bingham – plaster, paste

pseudoplastic – wet sand

tixotropic – paints, yoghurt

$$\nu = \frac{\eta}{\rho}$$ kinematic viscosity