# Seminary exercise Nr. 3
## Newton's laws and Conservation of energy

**4. The position vector of a mass particle is given by $x(t)=A+Bt^2$ and $y(t)=-Ct^2$, where $A$ , $B$ and $C$ are constants (explain their meaning in physics). Find the components of the velocity and acceleration vectors and calculate their magnitudes. Describe the type of motion.**

$x(t)=A+Bt^2$

$y(t)=-Ct^2$

$v_x=?$

$v_y=?$

$a_x=?$

$a_y=?$

$\|\vec{v}\|=?$

$\|\vec{a}\|=?$

$$v_x=\frac{dx}{dt}=\frac{d}{dt}\left(A+Bt^2\right)=2Bt \quad ; \quad v_y=\frac{dy}{dt}=\frac{d}{dt}\left(-Ct^2\right)=-2Ct$$

$$a_x=\frac{dv_x}{dt}=\frac{d}{dt}(2Bt)=2B \quad ; \quad a_y=\frac{dv_y}{dt}=\frac{d}{dt}(-2Ct)=-2C$$

$$\|\vec{v}\|=\sqrt{v_x^2+v_y^2}=\sqrt{(2Bt)^2+(-2Ct)^2}=\sqrt{4B^2t^2+4C^2t^2}=2t\sqrt{B^2+C^2}$$

$$\|\vec{a}\|=\sqrt{a_x^2+a_y^2}=\sqrt{(2B)^2+(-2C)^2}=\sqrt{4B^2+4C^2}=2\sqrt{B^2+C^2}$$

**8. A small ball was tossed vertically at a constant initial speed of $12\,ms^{-1}$ . Calculate the maximum theoretical height that can be reached. Use the law of conservation of energy.**

$v(0)=12\,ms^{-1}$

$x(0)=0\,m$

$g=9.81\,ms^{-2}$

$x(t_{max})=?$

$$t=0 \quad : \quad K(0)=\frac{1}{2}mv(0)^2 \quad ; \quad U(0)=0$$

$$t=t_{max} \quad : \quad K(t_{max})=0 \quad ; \quad U(t_{max})=mg\,x(t_{max})$$

$$K(0)+U(0)=K(t_{max})+U(t_{max}) \quad ; \quad \frac{1}{2}mv(0)^2=mg\,x(t_{max})$$

$$x(t_{max})=\frac{v(0)^2}{2g}=\frac{(12\,ms^{-1})^2}{2\cdot 9.81\,ms^{-2}}=7.34\,m$$

$$t \quad : \quad K(t)+U(t)=K(0) \quad ; \quad \frac{1}{2}mv(t)^2+mg\,x(t)=\frac{1}{2}mv(0)^2$$

$$v(t)^2+2g\,x(t)=v(0)^2$$

**9. A rescue plane flies at a constant speed of $200\,kmh^{-1}$ and height of $0.5\,km$ over the sea level. A rescue bag is dropped to fall down directly to the point of a victim location. What is the final impact speed of the bag? Use the law of conservation of energy.**

$v_x(0)=200\,km\;h^{-1}$
$=55.6\,ms^{-1}$

$y(0)=0.5\,km$
$=500\,m$

$y(t_{impact})=0\,m$

$x(0)=0\,m$

$g=9.81\,ms^{-2}$

$\|\vec{v}(t_{impact})\|=?$

$$K(0)=\frac{1}{2}mv_x(0)^2 \quad ; \quad U(0)=mg\,y(0)$$

$$K(t_{impact})=\frac{1}{2}m\left[\sqrt{v_x(t_{impact})^2+v_y(t_{impact})^2}\right]^2 \quad ; \quad U(t_{impact})=0$$

$$K(0)+U(0)=K(t_{impact})+U(t_{impact})$$

$$\frac{1}{2}mv_x(0)^2+mg\,y(0)=\frac{1}{2}m\left[v_x(t_{impact})^2+v_y(t_{impact})^2\right]$$

$$v_x(0)^2+2g\,y(0)=\left[v_x(t_{impact})^2+v_y(t_{impact})^2\right]$$

$$\|\vec{v}(t_{impact})\|=\sqrt{v_x(t_{impact})^2+v_y(t_{impact})^2}=\sqrt{v_x(0)^2+2g\,y(0)}=$$
$$=\sqrt{(55.6\,ms^{-1})^2+2\cdot 9.81\,ms^{-2}\,500\,m}=114\,ms^{-1}$$

**10. A block slides along a track from one level to a higher level after passing through an intermediate valley. The track is frictionless until the block reaches the higher level. Then a frictional force stops the block in a distance $d$ . The initial speed of the block is $6\,m\,s^{-1}$ , the height difference $1.1\,m$ , and $\mu_k=0.60$ . Find $d$ .**

$v(t_1)=6\,m\,s^{-1}$

$h_2-h_1=1.1\,m$

$\mu_k=0.60$

$d=?$

$$K(t_1)+U(t_1)=K(t_2)+U(t_2)$$

$$\frac{1}{2}m\,v(t_1)^2+m\,g\,h_1=\frac{1}{2}m\,v(t_2)^2+m\,g\,h_2$$

$$v(t_2)=\sqrt{v(t_1)^2-2g(h_2-h_1)}=\sqrt{(6\,m\,s^{-1})^2-2\cdot 9.81\,m\,s^{-2}\cdot 1.1\,m}=3.80\,m\,s^{-1}$$

$$\mu_k=\frac{F_f}{F_n}=\frac{F_f}{mg}\quad ;\quad F_f=\mu_k\,m\,g\quad ;\quad W_f=F_f\,d=\mu_k\,m\,g\,d=K(t_2)=\frac{1}{2}m\,v(t_2)^2$$

$$d=\frac{v(t_2)^2}{2\mu_k\,g}=\frac{(3.80\,m\,s^{-1})^2}{2\cdot 0.60\cdot 9.81\,m\,s^{-2}}=1.23\,m$$

**11. A diesel engine with a pulling force of $40\,kN$ accelerates a train from rest on a straight-line railway at constant acceleration of $0.5\,m\,s^{-2}$ . What is the total work done in $1\,min$ ?**

$F=40\,kN=$
$=4\cdot 10^4\,N$

$v(0)=0\,m\,s^{-1}$

$x(0)=0\,m$

$a=0.5\,m\,s^{-2}$

$t_1=1\,min=$
$=60\,s$

$W(t_1)=?$

$$x(t)=\frac{1}{2}a\,t^2\quad ;\quad x(t_1)=\frac{1}{2}a\,t_1^2=\frac{1}{2}\cdot 0.5\,m\,s^{-2}\cdot(60\,s)^2=900\,m$$

$$W(t_1)=F\left[x(t_1)-x(0)\right]=4\cdot 10^4\,N\cdot 900\,m=3.6\cdot 10^7\,J$$

$$v(t)=a\,t\quad ;\quad K(t)=\frac{1}{2}m\,v(t)^2=\frac{1}{2}m\,a^2t^2=\frac{1}{2}F\,a\,t^2$$

$$K(t_1)=W(t_1)=\frac{1}{2}F\,a\,t_1^2=\frac{1}{2}\cdot 4\cdot 10^4\,N\cdot 0.5\,m\,s^{-2}\cdot(60\,s)^2=3.6\cdot 10^7\,J$$

**12. A car of mass $1200\,kg$ moving at a constant speed of $100\,km\,h^{-1}$ starts to brake with a constant deceleration. Due to this, the car stops at a distance of $80\,m$ . Find the magnitude of the deceleration.**

$m=1200\,kg$

$v=100\,km\,h^{-1}=$
$=27.8\,m\,s^{-1}$

$d=80\,m$

$a=?$

$$K=\frac{1}{2}m\,v^2\quad ;\quad W=F\,d=m\,a\,d=K=\frac{1}{2}m\,v^2$$

$$a=\frac{v^2}{2\,d}=\frac{(27.8\,m\,s^{-1})^2}{2\cdot 80\,m}=4.83\,m\,s^{-2}$$

**13. A drop hammer of mass $500\,kg$ was dropped from a height of $1\,m$ . After it hits the formed material, the deceleration of the hammer takes $0.01\,s$ . Calculate the average forming force acting during the material deformation.**

$m=500\,kg$

$h=1\,m$

$\Delta t=0.01\,s$

$g=9.81\,m\,s^{-2}$

$F=?$

$$h=\frac{1}{2}g\,t_{fall}^2\quad ;\quad t_{fall}=\sqrt{\frac{2h}{g}}$$

$$v(t_{fall})=g\,t_{fall}=g\sqrt{\frac{2h}{g}}=\sqrt{2\,hg}=\sqrt{2\cdot 1\,m\cdot 9.81\,m\,s^{-2}}=4.43\,m\,s^{-1}$$

$$F=ma=m\frac{\Delta v}{\Delta t}=500\,kg\,\frac{4.43\,m\,s^{-1}}{0.01\,s}=2.22\cdot 10^5\,N$$

**14. A small cart of mass $m$ moves without sliding down on an incline that leads into a cylindrical loop of radius $r$. From what height $h$ must the cart go down to pass through the entire circular loop of the cylindrical surface? Neglect the moment of inertia and the rolling resistance of the wheels.**

$m$

$r$

$h=?$

At the highest point of the loop (top): $mg=ma=m\frac{v_{top}^2}{r}$ ; $v_{top}=\sqrt{gr}$

$$U_0=U_{top}+K_{top} \quad ; \quad mgh=mg\,2r+\frac{1}{2}mv_{top}^2=mg\,2r+\frac{1}{2}m\,gr \quad ; \quad h=\frac{5r}{2}$$