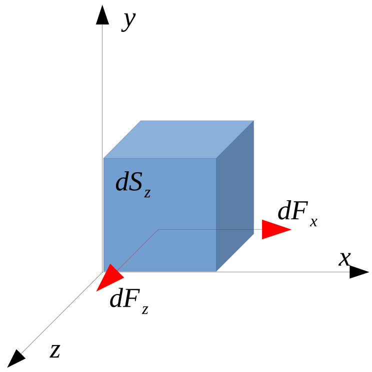
Seminary exercise Nr. 7
Continuum

1. Define a continuum material point (a differential volume) and describe its properties in x_1, x_2, x_3 coordinate system. For a surface force acting in the direction of selected axis, specify the existing stresses and define the stress subscripts.

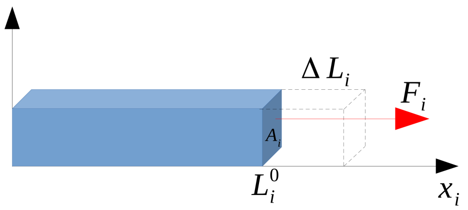


Shear stress: $\sigma_{zx} = \frac{dF_x}{dS_z}$; Normal stress: $\sigma_{zz} = \frac{dF_z}{dS_z}$

Stress tensor: $\boldsymbol{\sigma} = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{vmatrix}$

Conservation of angular momentum: $\sigma_{ij} = \sigma_{ji}$

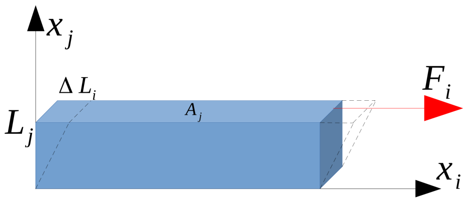
2. Define the Hooke's law for direct stress and shear stress. Specify the physical quantities including physical units. Describe the stress-strain curve.



Tensile strain: $\varepsilon_{ii}=\frac{\Delta L_i}{L_i^0}$ $[\cdot]$; Uniaxial stress: $\sigma_{ii}=\frac{F_i}{A_i}$ $[Pa]$

Elastic modulus: $E=\frac{\sigma_{ii}}{\varepsilon_{ii}}=\frac{F_i L_i^0}{A_i \Delta L_i}$ $[Pa]$

Hooke's law for direct stress: E is constants



Shear strain: $\varepsilon_{ij}=\frac{\Delta L_i}{L_j}$ $[\cdot]$; Shear stress: $\sigma_{ij}=\frac{F_i}{A_j}$ $[Pa]$

Shear modulus: $G=\frac{\sigma_{ij}}{\varepsilon_{ij}}=\frac{F_i L_j}{A_j \Delta L_i}$ $[Pa]$

Hooke's law for shear stress: G is constants

3. A mine shaft elevator is hanging on a steel rope ($E=2.1\cdot 10^{11}\,Pa$ **) with a diameter of** $2.5\,cm$ **. The total mass of the cabin and the transported people is** $650\,kg$ **. How does the steel rope extend when the lift is at a surface** $12\,m$ **below the motor of the elevator? How does the rope extend when the lift is at the bottom of a shaft** $350\,m$ **deep? Neglect the mass of the rope with respect to the mass of the cabin.**

$d=2.5\,cm=$
$=0.025\,m$
$m=650\,kg$
$h_1=12\,m$
$h_2=350\,m$
$E=2.1\cdot 10^{11}\,Pa$
$\Delta l_1=?$
$\Delta l_2=?$

$$E=\frac{F\,L}{A\,\Delta L} \quad ; \quad E=\frac{m\,g\,h}{\pi\left(\frac{d}{2}\right)^2 \Delta l}$$

$$\Delta l_1=\frac{m\,g\,h_1}{\pi\left(\frac{d}{2}\right)^2 E}=\frac{650\,kg\cdot 9.81\,m\,s^{-2}\cdot 12\,m}{\pi\left(\frac{0.025\,m}{2}\right)^2 2.1\cdot 10^{11}\,Pa}=0.000742\,m$$

$$\Delta l_2=\frac{m\,g\,h_2}{\pi\left(\frac{d}{2}\right)^2 E}=\frac{650\,kg\cdot 9.81\,m\,s^{-2}\cdot 350\,m}{\pi\left(\frac{0.025\,m}{2}\right)^2 2.1\cdot 10^{11}\,Pa}=0.0217\,m$$

6. A horizontal aluminium rod with a diameter of $4.8\,cm$ exits by $5.3\,cm$ from a wall. An object with mass of $1200\,kg$ is suspended from the end of the rod. The shear modulus of aluminium is $3\cdot10^{10}\,Pa$. Neglecting the mass of the rod, find the shear stress on the rod and the vertical deflection at the end of the rod.

$d=4.8\,cm=$
$=0.048\,m$

$l=5.3\,cm=$
$=0.053\,m$

$G=3\cdot10^{10}\,Pa$

$m=1200\,kg$

$\sigma=?$

$\Delta h=?$

$$G=\frac{F\,L}{A\,\Delta L}=\frac{\sigma L}{\Delta L}=\frac{m\,g\,l}{\pi\left(\frac{d}{2}\right)^2\Delta h}$$

$$\Delta h=\frac{m\,g\,l}{\pi\left(\frac{d}{2}\right)^2 G}=\frac{1200\,kg\cdot9.81\,m\,s^{-2}\cdot0.053\,m}{\pi\left(\frac{0.048\,m}{2}\right)^2 3\cdot10^{10}\,Pa}=1.15\cdot10^{-5}\,m$$

$$\sigma=\frac{G\,\Delta L}{L}=\frac{G\,\Delta h}{l}=\frac{3\cdot10^{10}\,Pa\cdot1.15\cdot10^{-5}\,m}{0.053\,m}=6.51\cdot10^{6}\,Pa$$