Seminary exercise Nr. 10

Thermodynamics I – Heat capacity, Equation of state, Kinetic theory

For all exercises, let assume the following values:

- **Avogadro's constant** $N_{Av}=6.022\cdot 10^{23} mol^{-1}$
- **gas constant** $R=8.31\,J\,mol^{-1}K^{-1}$

1. A certain substance of mass $5\,kg$ has to be warmed up by $10\,°C$. This process required $200\,kJ$ of heat. Determine the heat capacity and the specific heat capacity of the substance.

$m=5\,kg$

$\Delta T=10\,°C=$
$=10\,K$

$Q=200\,kJ=$
$=2\cdot 10^5\,J$

$C=?$

$c=?$

$$C=\frac{Q}{\Delta T}=\frac{2\cdot 10^5\,J}{10\,K}=2\cdot 10^4\,J\,K^{-1}$$

$$c=\frac{Q}{m\Delta T}=\frac{2\cdot 10^5\,J}{5\,kg\cdot 10\,K}=4\cdot 10^3\,J\,kg^{-1}K^{-1}$$

4. A mass of $20\,kg$ was hanged on a vertically fixed steel wire of a length $1\,m$ and diameter of $2\,mm$. What temperature change would compensate the wire extension? The linear thermal expansion coefficient of steel is $\alpha=1.2\cdot 10^{-5}K^{-1}$ and its elastic modulus is $E=2.1\cdot 10^{11}\,Pa$.

$m=20\,kg$

$l_0=1\,m$

$d=2\,mm=$
$=0.002\,m$

$\alpha=1.2\cdot 10^{-5}K^{-1}$

$E=2.1\cdot 10^{11}Pa$

$\Delta T=?$

$$E=\frac{F\,l_0}{A\,\Delta l}\quad ;\quad \Delta l=\frac{m\,g\,l_0}{\pi\left(\frac{d}{2}\right)^2 E}=\frac{20\,kg\cdot 9.81\,m\,s^{-2}\cdot 1\,m}{\pi\cdot\left(\frac{0.002\,m}{2}\right)^2\cdot 2.1\cdot 10^{11}\,Pa}=2.97\cdot 10^{-4}\,m$$

$$\alpha=\frac{\Delta l}{l_0\,\Delta T}\quad ;\quad \Delta T=\frac{\Delta l}{l_0\,\alpha}=\frac{2.97\cdot 10^{-4}\,m}{(1\,m+2.97\cdot 10^{-4}\,m)\cdot 1.2\cdot 10^{-5}\,K^{-1}}=24.8\,K$$

5. A sample of pure titanium ^{48}Ti has a mass $50\,g$. Determine the number of atoms and moles in the sample.

^{48}Ti

$m=50\,g$

$AM=48\,g\,mol^{-1}$

$N=?$

$n=?$

$$n=\frac{m}{AM}=\frac{50\,g}{48\,g\,mol^{-1}}=1.042\,mol$$

$$N=n\,N_{Av}=1.042\,mol\cdot 6.022\cdot 10^{23}\,mol^{-1}=6.27\cdot 10^{23}\,atoms$$

8. Determine the volume of a pressure vessel filled by $100\,g$ of oxygen at $200\,kPa$ of pressure and temperature of $25\,°C$. Consider that the molecular mass of oxygen is $32\,g\,mol^{-1}$.

$m=100\,g$

$p=200\,kPa=$
$=2\cdot10^5\,Pa$

$T=25\,°C=$
$=298\,K$

$MM=32\,g\,mol^{-1}$

$V=?$

$$pV=nRT$$

$$V=\frac{nRT}{p}=\frac{\frac{m}{MM}RT}{p}=\frac{\frac{100\,g}{32\,g\,mol^{-1}}\cdot8.31\,J\,mol^{-1}\,K^{-1}\cdot298\,K}{2\cdot10^5\,Pa}=0.0387\,m^3$$

10. Determine the specific heat capacity at constant volume c_V and the specific heat capacity at constant pressure c_p for pure gaseous hydrogen.

H_2

$c_V=?$

$c_p=?$

$$c_V=\frac{5}{2}R=\frac{5}{2}\cdot8.31\,J\,mol^{-1}K^{-1}=20.78\,J\,mol^{-1}\,K^{-1}$$

$$c_p=c_V+R=\frac{7}{2}R=\frac{7}{2}\cdot8.31\,J\,mol^{-1}K^{-1}=29.08\,J\,mol^{-1}K^{-1}$$

12. What is the internal energy of $5\,kg$ of gaseous oxygen at $25\,°C$? How does the internal energy change after the gas is warmed to $100\,°C$ at constant volume? Consider that the molecular mass of oxygen is $32\,g\,mol^{-1}$.

O_2

$m=5\,kg=$
$=5000\,g$

$T_1=25\,°C=$
$=298\,K$

$MM=32\,g\,mol^{-1}$

$U_1=?$

$T_2=100\,°C=$
$=373\,K$

$\Delta U=?$

$$U_1=\frac{5}{2}nRT_1=\frac{5}{2}\frac{m}{MM}RT_1=\frac{5}{2}\frac{5000\,g}{32\,g\,mol^{-1}}\cdot8.31\,J\,mol^{-1}K^{-1}\cdot298\,K=9.67\cdot10^5\,J$$

$$U_2=\frac{5}{2}\frac{m}{MM}RT_2=\frac{5}{2}\frac{5000\,g}{32\,g\,mol^{-1}}\cdot8.31\,J\,mol^{-1}\,K^{-1}\cdot373\,K=1.21\cdot10^6\,J$$

$$\Delta U=U_2-U_1=1.21\cdot10^6\,J-9.67\cdot10^5\,J=2.43\cdot10^5\,J$$

$$\Delta U=Q=nc_V\Delta T=n\frac{5}{2}R(T_2-T_1)=\frac{m}{MM}\frac{5}{2}R(T_2-T_1)=$$

$$=\frac{5000\,g}{32\,g\,mol^{-1}}\cdot\frac{5}{2}\cdot8.31\,J\,mol^{-1}\,K^{-1}\cdot(373\,K-298\,K)=2.43\cdot10^5\,J$$