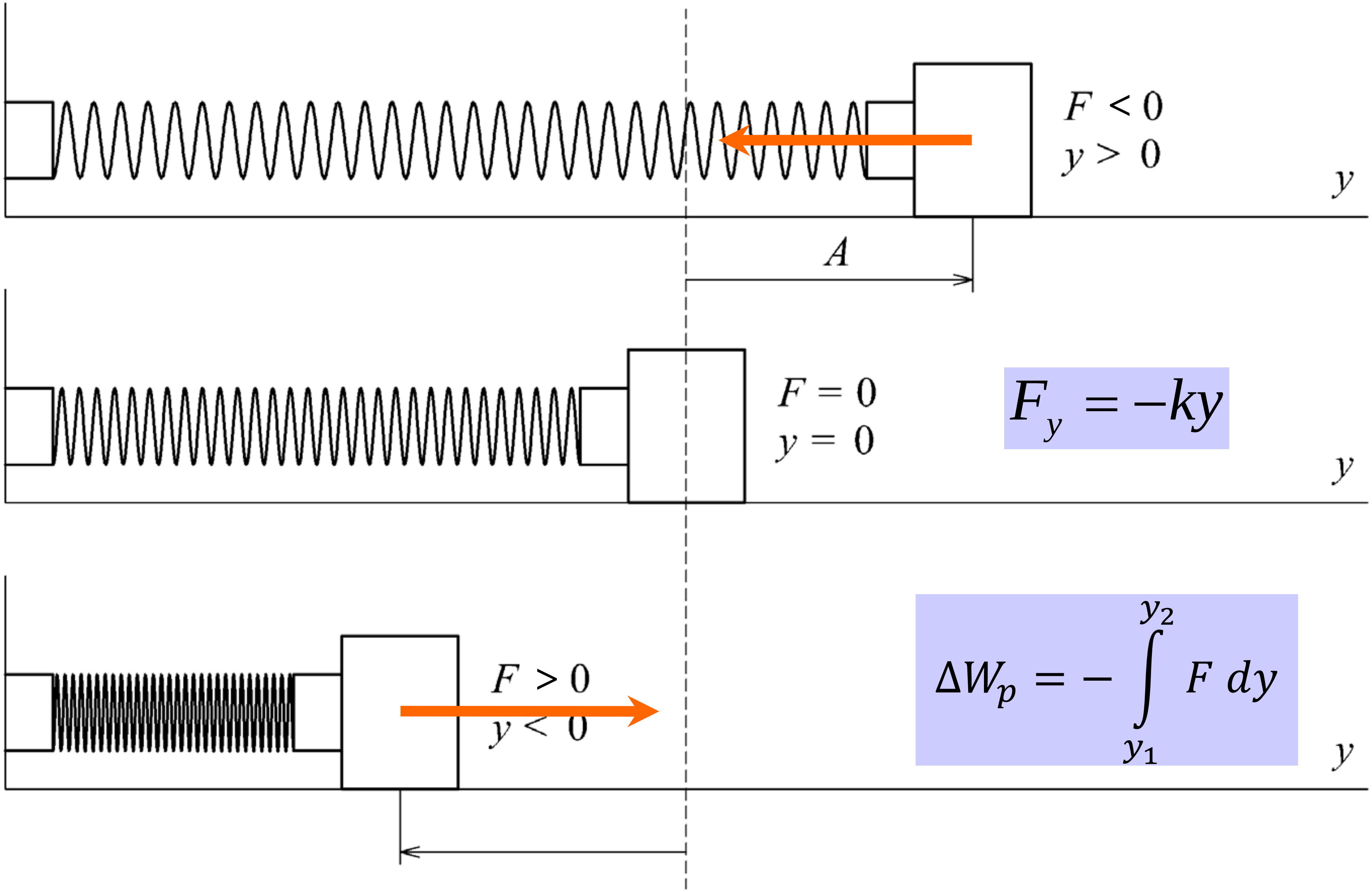
Harmonic motion



$$F_y = -ky$$

$$\Delta W_p = -\int_{y_1}^{y_2} F\,dy$$

Free undamped oscillations

only elastic (reaction) force exists → constant amplitude

$$F_y = -ky$$

$$ma_y = m\frac{d^2y}{dt^2} = -ky$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0 \qquad \frac{k}{m} = \omega^2$$

$$\ddot{y} + \omega^2 y = 0$$

solution $y = A\cos(\omega t + \varphi_0)$

$$y = A\sin(\omega t + \varphi_0)$$

characteristics: stiffness, angular frequency, period, phase

displacement of amplitude, velocity and acceleration

initial phase

Free damped oscillations

elastic force + damping force → decreasing amplitude in time

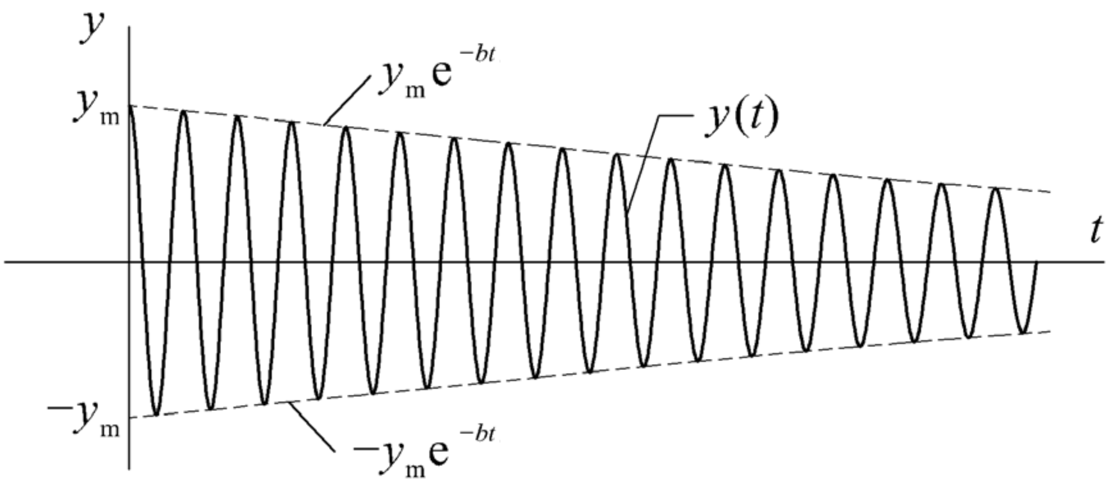
$$ma = m\frac{d^2y}{dt^2} = -ky - B\frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = 0$$

$$b = \frac{B}{2m}$$ damping constant

$$\ddot{y} + 2b\dot{y} + \omega^2 y = 0$$

solution $y = Ae^{-bt}\sin\left(\omega_1 t + \varphi_0\right)$

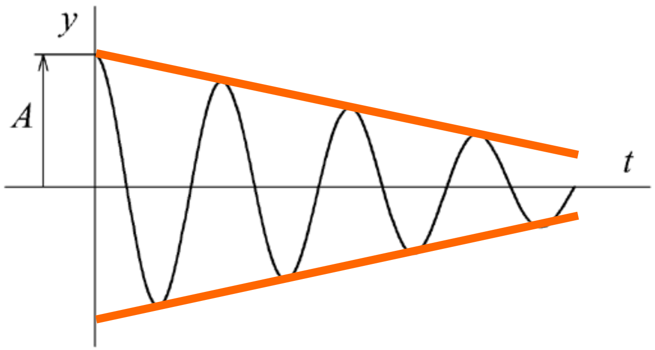


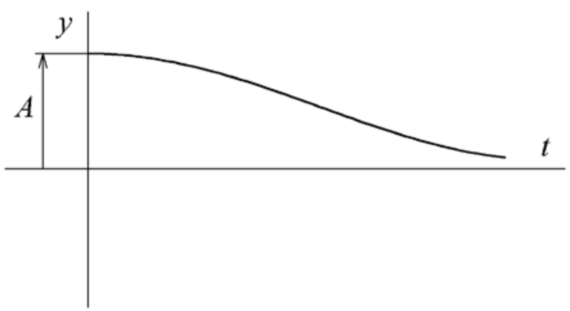
characteristics: damping ratio $\beta = \frac{y_i}{y_{i+1}} = e^{bT}$

logarithmic decrement $\delta = \ln\beta = bT$

angular velocity $\omega_1 = \sqrt{\omega_0^2 - b^2}$

period $T = \frac{2\pi}{\sqrt{\omega_0^2 - b^2}}$





Forced oscillations

elastic force + damping force + excitation by a harmonic force

$$ma = m\frac{d^2y}{dt^2} = -ky - B\frac{dy}{dt} + F_0 \sin \Omega t$$

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = \frac{F_0}{m}\sin \Omega t$$

solution $y = y_h + y_p$

$$y_h = A_h e^{-bt} \sin(\omega_t t + \varphi_0)$$

$$y_p = A_v \sin(\Omega t + \Psi_0)$$

$$A_v^2 = \frac{\left(\frac{F_0}{m}\right)^2}{4b^2\Omega^2 + (\omega_0^2 - \Omega^2)^2}$$

resonance of amplitude $\boldsymbol{\Omega_r = \sqrt{\omega_0^2 - 2b^2}}$