Experiment Nr. 10

DETERMINATION OF YOUNG'S MODULUS FROM WIRE EXTENSION

Theoretical part

All solid bodies tend to change their dimensions and deform due to mechanical loading. If the applied forces do not exceed definitely high values, the dependence of the deformation and the load could be linear. If the acting force direction is coincident to the thin wire (rod) longitudinal axis, the extension can be defined as

$$\frac{\Delta l}{l_0} = \frac{F}{S}\frac{1}{E} = \frac{\sigma}{E} = const.$$

where $\frac{\Delta l}{l}$ is the relative extension of a wire with a length l_0 and cross-section area S caused by acting force F. The quantity σ is direct stress, $[\sigma]$ = Pa, and E is the Young's modulus of elasticity, [E] = Pa. The afore-mentioned formula is called Hooke's law and its validity is limited by the proportionality limit, thus, for relatively low strains in the elastic deformation range.

The Hooke's law can be used for both extension and depression. However, some of the materials (concrete, cast iron, etc.) show a heavy hysteresis, thus, the extension modulus tend to differ from depression modulus. Secondly, the elastic modulus could show a dependency on the temperature. The Young's moduli of selected materials are arranged in the following table:

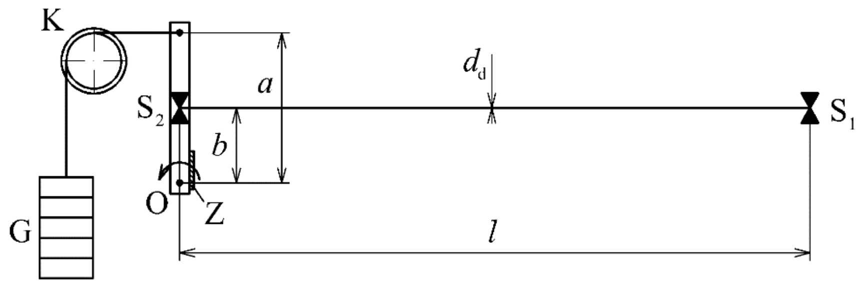
material	***E* [Pa]**
steel	$2{,}1\cdot10^{11}$
aluminum	$(6{,}6–6{,}8)\cdot10^{10}$
copper	$(1{,}2–1{,}3)\cdot10^{11}$
brass	$(1{,}0–1{,}7)\cdot10^{11}$
duralumin	$7{,}2\cdot10^{10}$

In our case, we will use a direct Young's modulus measurement method - a thin wire is directly loaded and the relative extension is recorded. Prior the measurement, the initial wire length l_0 and its cross-section area S has to be determined. During the measurement, the displacement Δl as a function of acting force F is recorded. Small changes of the cross-section area could be neglected.

The extension range where the Hooke's law is valid is quite small (about 1-2 % of the relative extension), thus, a special observation methods have to be used. In our measurement, we will use the laser pointer beam reflected on a small mirror fixed on the lever mechanism connected with the loaded wire. The displacement of the wire is magnified by the angular motion of the mirror and the laser spot position could be read on a ruler placed in a far distance R.

Theory of the Young's modulus determination

The measuring procedure considers the wire extension within Hooke's law validity range. The measurement scheme could be seen in the following figure:



The end of the wire S_1 is fixed to the instrument frame. Another end S_2 is connected to a lever mechanism changing the linear displacement of the wire to the angular displacement of the lever. The centre of the rotation axis O is shifted off the longitudinal wire axis. The wire loading is created by several mass bodies placed on a vertical holder. Considering the geometry of the lever, the force *F* acting on the wire is

$$F = G\frac{a}{b},$$

where *G* is the gravitational force of the mass body and *a*, *b* are the lever dimensions (see the figure above).
The acting force *F* causes the wire extension Δl. Due to this, the lever moves with an angular displacement $\Delta\varphi$. For small displacements, the extension is

$$\Delta l = l - l_0 = b\Delta\varphi$$

For the extension reading, we will use the laser pointer beam reflected on a small mirror fixed on the lever mechanism. The displacement of the wire is magnified by the angular motion of the mirror and the laser spot position could be read on a ruler placed in a far distance *R*. Considering this, the change of the laser spot position Δn can be used for the real wire extension calculation:

$$\Delta l = b\frac{\Delta n}{2R}$$

Using the Hooke's law, the Young's modulus can be calculated by

$$E = \frac{2Rl_0 a}{Sb^2}\frac{G}{\Delta n}$$

For unloaded wire, the laser spot on the ruler is at the position n_0. Increasing the load by several mass bodies of a weight G_i we obtain corresponding laser spot positions n_i. The afore-mentioned formula can be modified to

$$E = \frac{2Rl_0 a}{Sb^2}\frac{1}{K}$$

The coefficient *K* should be determined as the slope coefficient of the linear function given by

$$n_i = n_0 + KG_i$$

Measurement objectives

1. Plot the function $n_i = n_0 + KG_i$ for sufficient number of points.

2. Determine the Young's modulus of the wire and evaluate the measurement uncertainty. Compare the obtained result to the material property given by the tabular data.

Measurement procedure

Determine the weight (in Newtons) of all available mass bodies using a precise digital scale. Determine the wire diameter (check the value in several positions along the wire) and its length. Load the wire stepwise by adding the mass bodies and record the appropriate laser spot positions on the ruler. Subsequently, unload the wire stepwise by removing the mass bodies in the reverse order and record the appropriate laser spot positions on the ruler again. Calculate the average spot positions for particular load values and prepare the data for the linear function $n_i = n_0 + KG_i$. Using the linear regression, calculate the slope coefficient *K*.

Important constants

The lever dimension are: a = (90,00 ± 0,50) mm
b = (50,00 ± 0,50) mm

Young's modulus uncertainty calculation notes

The combined uncertainty consist of both Type A and Type B uncertainty. The relative Type A uncertainty can be determined by

$$u_{rEA} = \sqrt{u_{rSA}^2 + u_{rKA}^2},$$

where u_{rSA} is the relative uncertainty of the wire cross-section determined using standard deviation of the diameter measurement data (micrometre precision) and u_{rKA} is the slope coefficient uncertainty given by the regression calculation results.
The relative Type B uncertainty can be calculated from the Type B sub uncertainties of the *S*, *R*, l_0, *a* and *b* values as follows:

$$u_{rEB} = \sqrt{u_{rSB}^2 + u_{rRB}^2 + u_{rl_0B}^2 + u_{raB}^2 + 4u_{rbB}^2}$$

The combined standard uncertainty of the Young's modulus is

$$u_{rE} = \sqrt{u_{rEA}^2 + u_{rEB}^2}$$