Experiment Nr. 27

DETERMINATION OF THE ACOUSTIC WAVE FREQUENCY USING INTERFERENCE

Theoretical part

Acoustic waves (sound) can propagate within a material substance called a medium (solid, liquid, gaseous state and plasma). A sound wave is a travelling longitudinal elastic wave, thus, it causes a periodic compression and dilution of the medium. Near a point source, the wave-fronts are spherical and are spreading out in three dimensions (spherical waves). As the wave-fronts move outward and their radii become larger, their curvatures decrease. Far from the source they can be assumed to be planar.
The human ear is sensitive to sound waves in the frequency range from about 16 Hz to 20 kHz. The sound waves with frequencies higher than 20 kHz are called as ultrasound. The sound waves with frequencies lower than 16 Hz are called as infrasound.
The simplest sound waves are sinusoidal waves with definite frequency, amplitude and wavelength. A sinusoidal sound wave in an elastic medium is described by the following wave function

$$u = u_0 \sin \omega \left(t - \frac{x}{c} \right) \qquad (1)$$

where u_0 is the wave amplitude, ω is the angular velocity, t is time and c is the phase velocity of the wave in the selected medium. The wavelength λ and the wave frequency f define the phase velocity by following formula

$$\lambda = \frac{c}{f} \qquad (2)$$

Sound waves can also be described in terms of the air pressure variation. The pressure fluctuates above and below the atmospheric pressure. Its variation in time is sinusoidal and has the same angular frequency ω as that of the motion of air particles. The sound pressure variations are given by

$$p = p_0 \cos(t - \frac{x}{c}) \text{ , } [p] = \text{Pa} \qquad (3)$$

Thus the relation between the pressure and the sound wave function is

$$p = \rho v c \qquad (4)$$

where ρ is the medium density and v is the sound particle velocity (it's the velocity of the wave oscillations, not the phase velocity !). The pressure amplitude for the lowest detectable sound intensity at the frequency 1 kHz is about $2.8 \cdot 10^{-5}$ Pa. The root-mean-square (RMS or effective) value of the pressure is

$$p_{ef} = \frac{p_0}{\sqrt{2}} \tag{5}$$

The maximum pressure amplitude p_0 of a sound wave that the human ear can tolerate is about 28 Pa (much less than the normal atmospheric pressure of about 10^5 Pa).
The intensity I of a travelling wave is defined as the time average rate at which energy is transported by the wave, per unit area, across a surface perpendicular to the direction of propagation. Briefly, the intensity is the average power transported per unit area that is perpendicular to the direction of the wave propagation. The intensity of as sound wave can be expressed as

$$I = \frac{p_{ef}^2}{\rho c}, \; [I] = \mathrm{W \cdot m^{-2}} \tag{6}$$

where the speed of sound c is temperature sensitive according to the relation (temperature t is in °C and speed in $\mathrm{m.s^{-1}}$)

$$c = 344.3 + 0.62\,(t - 20) \tag{7}$$

Because of extremely large range of intensities over which the human ear is sensitive (up to 12 orders of magnitude), a logarithmic rather than arithmetic intensity scale is convenient. The intensity level L_I of a sound wave is defined by

$$L_I = 10 \log \frac{I}{I_0}, \; [L_I] = \mathrm{dB} \tag{8}$$

where I_0 is the threshold of hearing intensity: $I_0 = 1.10^{-12}\ \mathrm{W.m^{-2}}$.

Theory of the measurement principle

Ultrasonic waves of the same frequency, amplitude and direction of propagation are generated by two sources positioned parallel to each other (Fig. 1). If specific conditions are met, a phenomenon called interference could be observed. The interference of waves means a space superposition of the two or more wave functions. The wave sources can interfere at different phase positions. If one of the sources is shifted (in-phase and out-of-phase typically), the interference pattern could be obtained as well. The angular distribution of the intensity of the waves, which interfere with each other, is automatically recorded using a motor-driven ultrasonic detector and a PC. The computer cannot observe the wave intensity directly, the voltage as the detector response to the wave intensity is recorded.
Two identical sources of sound S_1 and S_2 are at a distance of $2d$ from each other and emit waves of the same frequency and phase perpendicularly to their connecting line. When the path difference $\Delta\vec{r} = \vec{r}_1 - \vec{r}_2$ of the two waves at a point P is a multiple of the source wavelength λ, then the waves are subject to constructive interference. Should $\Delta\vec{r}$ be an uneven multiple of $\frac{\lambda}{2}$, however, then they will interfere destructively with each other.

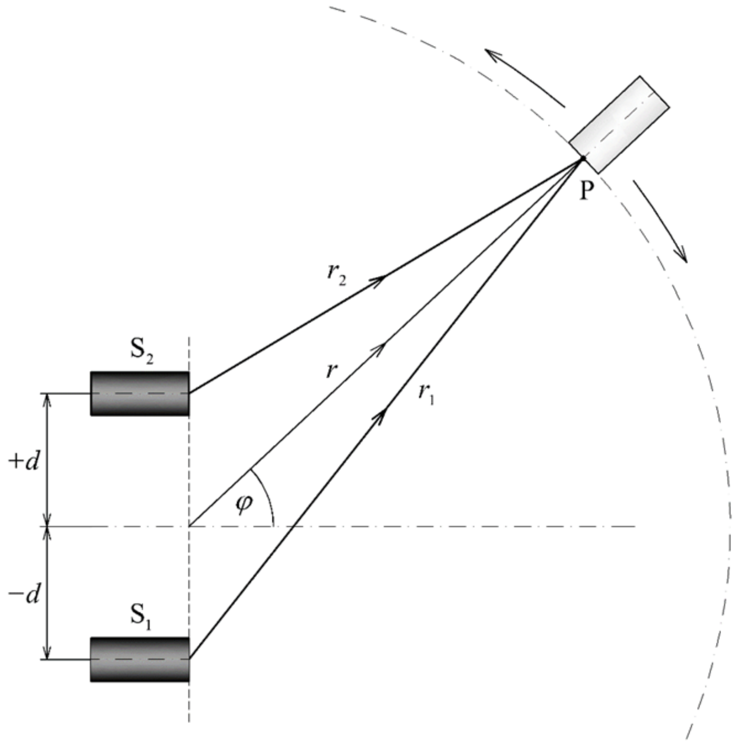


Fig. 1 The measurement principle

Using cosine theorem we can determine the r_1 path as

$$r_1 = \sqrt{r^2 + d^2 - 2dr\cos\left(\frac{\pi}{2}+\varphi\right)} = r\sqrt{1+\frac{d^2}{r^2} - 2\frac{d}{r}\cos\left(\frac{\pi}{2}+\varphi\right)} = \\ = r\sqrt{1+\frac{d^2}{r^2}+2\frac{d}{r}\sin\varphi} \tag{9}$$

An equivalent formula could be defined for the r_2 path. As $r >> d$, the term $\frac{d^2}{r^2}$ in the root can be neglected. Considering this we can get

$$r_1 \approx r\sqrt{1-2\frac{d}{r}\sin\varphi} \approx r - d\sin\varphi \tag{10}$$

The second approximation in equation (9) is also purposeful. On squaring the right side of equation (3), it is found that both terms are only then approximately equal when (r^2- $2dr \sin \varphi$) >> $d^2 \sin^2 \varphi$. This is fulfilled under our measurement setup (r = 55 cm, d < 10 cm and φ < 60°), however.
Analogously, for r_2

$$r_2 = r + d\sin\varphi \tag{11}$$

We thus have for the path difference:

$$\Delta r = 2d\sin\varphi \tag{12}$$

From which it follows for the angle of the maxima

$$\varphi_{\max} = \arcsin\left(n\frac{\lambda}{2d}\right), \quad n = 0, 1, 2, 3, \ldots \tag{13}$$

In the direction of the middle axis (φ = 0°), both partial waves always have the same path (there is no path difference), so that an intensity maximum must always be given here.
The minima lie at the angles:

$$\varphi_{\min} = \arcsin\left(\frac{2m+1}{2}\frac{\lambda}{2d}\right), \quad m = 0, 1, 2, 3, \ldots \tag{14}$$

The interference pattern of the two ultrasonic sources when these are in opposite phase operation, i.e. with a phase difference of π (which corresponds to the path difference of $\frac{\lambda}{2}$), is different. The geometric path difference Δr is now supplemented by the value of $\frac{\lambda}{2}$. This has the result that maxima are changed to minima and minima to maxima. In particular, a minimum is always given at φ = 0°. For the evaluation of the extremes, equations (13) and (14) must now be used the other way round:

$$\varphi'_{\max} = \arcsin\left(\frac{2m+1}{2}\frac{\lambda}{2d}\right), \quad m = 0, 1, 2, 3, \ldots \tag{15}$$

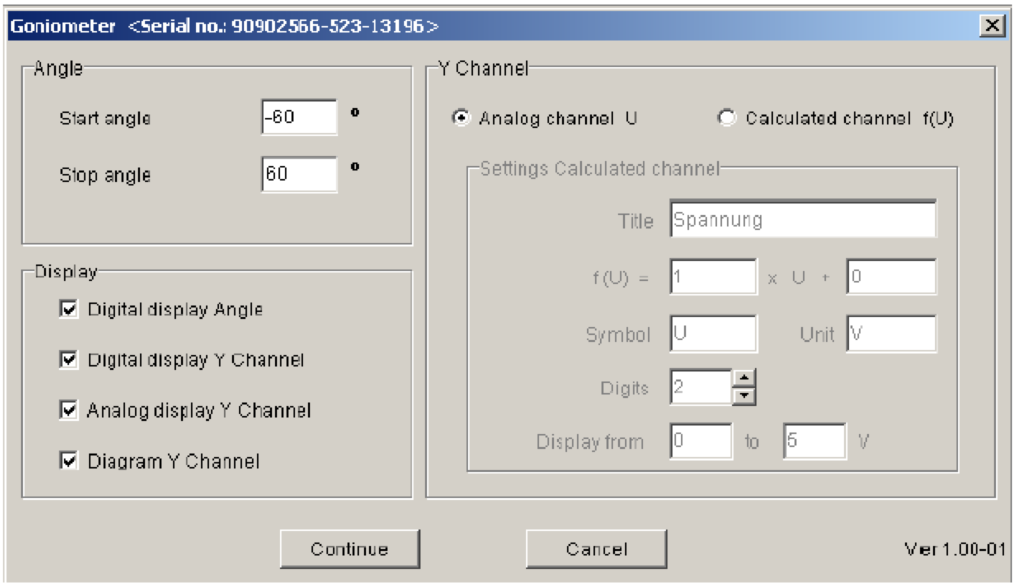
$$\varphi'_{\min} = \arcsin\left(n\frac{\lambda}{2d}\right), \quad n = 0, 1, 2, 3, \ldots \tag{16}$$

Measurement objectives

1. Determine the angular positions of the interference extremes of two ultrasonic transmitters vibrating in-phase.
2. Determine the angular positions of the interference extremes of two ultrasonic transmitters vibrating out-of-phase.
3. Analyse the positions of minima and maxima in all measurements and calculate the frequency of the acoustic waves. Compare to the results to the theoretical value.

Measurement procedure

First, check the position of the goniometer that should be set to φ = 0°. Switch on the power supplies and run the software "Measure". Check the parameters of the measurement according to the following figure:



Set one of the transmitters to the position "in-phase" and start the first measurement. After the data acquisition is finished, go to the start position of the detector, change the transmitter to the "out-of-phase" position and start the second measurement. After finishing the data acquisition, calculate the positions of maxima and minima of both obtained curves (use the function "Curve analysis" in "Analysis" menu). Export the results and separate the data into four independent columns (in-phase maxima, in-phase minima, out-of-phase maxima and out-of-phase minima). Prepare four regressions of linear function $\sin\varphi_n = f(n)$, where n is the value number of order. Determine the slope coefficients a_1, a_2, a_3, and a_4 that correspond to the wavelengths λ_1, λ_2, λ_3, λ_4 by following formula:

$$\lambda_i = 2d\, a_i,$$

where d is the sources distance and a_i is the particular slope coefficient. Compare the obtained values and in case of good data homogeneity calculate the mean value λ. Using formula (2) determine the frequency of the ultrasound (be careful on correct determination of the sound wave speed in air being dependent on the air temperature). The estimated value should be within a range of (39 - 41) kHz.

Uncertainty calculation notes

No determination of the uncertainty is required. Compare your result to the theoretical frequency of the ultrasound and try to explain possible result deviation.