

Seminary exercise Nr. 4

Oscillations

1. The displacement of a particle oscillating in a simple undamped harmonic motion is defined by the function $x(t) = A \cos(\omega t + \varphi_0)$. Define the frequency, the period and the phase of the oscillations. Find the expression of the particle velocity and acceleration.
2. In an electric shaver, the blade moves back and forth over a distance of 2 mm in a simple undamped harmonic motion, with a frequency of 120 Hz . Find the amplitude, the maximum blade speed, and the magnitude of the maximum blade acceleration.
3. A body with mass of 0.12 kg undergoes a simple harmonic motion of amplitude 8.5 cm and period 0.2 s . What is the magnitude of the maximum force acting on it? If the oscillations are produced by a spring, what is the spring constant? What is the mechanical energy of this system?
4. A block rides on a piston (a squat cylindrical piece) that is moving vertically in a simple harmonic motion. If the harmonic motion has period of 1 s , at what amplitude of the motion will the block and the piston separate? If the motion has an amplitude of 5 cm , what is the maximum frequency for which the block and the piston will be in contact continuously?
5. What is the maximum acceleration of a platform that oscillates with an amplitude of 2.2 cm and a frequency of 6.6 Hz ?
6. A block is oscillating at the end of a spring and on a well-lubricated horizontal track. Suppose that the block has a mass $m = 2.72 \cdot 10^5 \text{ kg}$ and is designed to oscillate at a frequency $f = 10 \text{ Hz}$ and with an amplitude of 20 cm . What is the speed of the block as it passes through the equilibrium point? What is the total mechanical energy E of the spring-block system?
7. A damped oscillator is made of a block oscillating at the end of a spring. Suppose that the mass of the block is $m = 250 \text{ g}$, the spring constant is $k = 85 \text{ N m}^{-1}$ and the damping constant of the spring is $b = 0.14 \text{ s}^{-1}$. What is the period of the motion? How long does it take for the amplitude of the damped oscillations to drop to half of its initial value?
8. The amplitude of a lightly damped oscillator decreases by 3% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?
9. An external harmonic force is acting on a damped steel string. The frequency of the acting force is Ω and the string displacement due to oscillations is described by the formula $x(t) = A \sin(\Omega t + \varphi)$. The amplitude of the forced oscillations is given by
$$A = \frac{C}{\sqrt{(\omega^2 - \Omega^2)^2 + 4b^2\Omega^2}}$$
, where $\omega = 50 \text{ rad s}^{-1}$ is the natural frequency of the string, $b = 0.5 \text{ s}^{-1}$ is the damping constant and C is a real constant. Find the frequency of the acting force in resonance.