Seminary exercise Nr. 4

Oscillations

2. In an electric shaver, the blade moves back and forth over a distance of $2\,mm$ in a simple harmonic motion, with a frequency of $120\,Hz$. Find the amplitude, the maximum blade speed, and the magnitude of the maximum blade acceleration.

$d=2\,mm=$
$=2\cdot10^{-3}\,m$
$f=120\,Hz$
$A=?$
$v_{max}=?$
$a_{max}=?$

$$x=A\cos(\omega t+\varphi)\ ;\quad x_{max}=A\ ;\quad x_{min}=-A\ ;\quad d=2A$$

$$A=\frac{d}{2}=\frac{2\cdot10^{-3}\,m}{2}=1\cdot10^{-3}\,m\ ;\quad \omega=2\pi f=2\pi\cdot120\,Hz=754\,rad\,s^{-1}$$

$$\varphi:=0\ ;\quad x=A\cos(\omega t)\ ;\quad v=-A\,\omega\sin(\omega t)$$

$$v_{max}=A\,\omega=1\cdot10^{-3}\,m\cdot754\,rad\,s^{-1}=0.754\,m\,s^{-1}$$

$$a=-A\,\omega^2\cos(\omega t)\ ;\quad a_{max}=A\,\omega^2=1\cdot10^{-3}\,m\cdot(754\,rad\,s^{-1})^2=569\,m\,s^{-2}$$

3. A body with mass of $0.12\,kg$ undergoes a simple harmonic motion of amplitude $8.5\,cm$ and period $0.2\,s$. What is the magnitude of the maximum force acting on it? If the oscillations are produced by a spring, what is the spring constant? What is the mechanical energy of this system?

$m=0.12\,kg$
$A=8.5\,cm=$
$=8.5\cdot10^{-2}\,m$
$T=0.2\,s$
$F_{max}=?$
$k=?$
$E=?$

$$x=A\cos(\omega t)\ ;\quad v=-A\,\omega\sin(\omega t)\ ;\quad a=-A\,\omega^2\cos(\omega t)$$

$$\omega=\frac{2\pi}{T}=\frac{2\pi}{0.2\,s}=31.4\,rad\,s^{-1}\ ;\quad a_{max}=A\,\omega^2$$

$$F_{max}=m\,a_{max}=m\,A\,\omega^2=0.12\,kg\cdot8.5\cdot10^{-2}\,m\cdot(31.4\,rad\,s^{-1})^2=10.1\,N$$

$$\omega=\sqrt{\frac{k}{m}}\ ;\quad k=\omega^2 m=(31.4\,rad\,s^{-1})^2\cdot0.12\,kg=118\,N\,m^{-1}$$

$$E=V+K=\frac{1}{2}k\,x^2+\frac{1}{2}m\,v^2=\frac{1}{2}k[A\cos(\omega t)]^2+\frac{1}{2}m[-A\,\omega\sin(\omega t)]^2=$$

$$=\frac{1}{2}k\,A^2\cos^2(\omega t)+\frac{1}{2}m\,A^2\,\omega^2\sin^2(\omega t)=\frac{1}{2}k\,A^2\cos^2(\omega t)+\frac{1}{2}m\,A^2\,\frac{k}{m}\sin^2(\omega t)=$$

$$=\frac{1}{2}k\,A^2[\cos^2(\omega t)+\sin^2(\omega t)]=\frac{1}{2}k\,A^2=\frac{1}{2}118\,N\,m^{-1}\cdot(8.5\cdot10^{-2}\,m)^2=0.426\,J$$

4. A block rides on a piston (a squat cylindrical piece) that is moving vertically in a simple harmonic motion. If the harmonic motion has period of $1\,s$, at what amplitude of the motion will the block and the piston separate? If the motion has an amplitude of $5\,cm$, what is the maximum frequency for which the block and the piston will be in contact continuously?

$T=1\,s$
$A=?$
$g=9.81\,m\,s^{-2}$
$A'=5\,cm=$
$=5\cdot10^{-2}\,m$
$f_{max}=?$

$$\omega=\frac{2\pi}{T}=\frac{2\pi}{1\,s}=6.28\,rad\,s^{-1}\ ;\quad x=A\cos(\omega t)\ ;\quad a=-A\,\omega^2\cos(\omega t)$$

$$a_{max}=A\,\omega^2=g\ ;\quad A=\frac{g}{\omega^2}=\frac{9.81\,m\,s^{-2}}{(6.28\,rad\,s^{-1})^2}=0.249\,m\ ;\quad a_{max}=g=A'\,\omega_{max}^2$$

$$\omega_{max}=\sqrt{\frac{g}{A'}}=\sqrt{\frac{9.81\,m\,s^{-2}}{5\cdot10^{-2}\,m}}=14.0\,rad\,s^{-1}\ ;\quad f_{max}=\frac{\omega_{max}}{2\pi}=\frac{14.0\,rad\,s^{-1}}{2\pi}=2.23\,Hz$$

6. A block is oscillating at the end of a spring and on a well-lubricated horizontal track. Suppose that the block has a mass $m=2.72\cdot10^5 kg$ **and is designed to oscillate at a frequency** $f=10\,Hz$ **and with an amplitude of** $20\,cm$ **. What is the speed of the block as it passes through the equilibrium point? What is the total mechanical energy** E **of the spring-block system?**

$m=2.72\cdot10^5 kg$

$f=10\,Hz$

$A=20\,cm=$
$=0.2\,m$

$v_{eq}=?$

$E=?$

$$\omega=2\pi f=2\pi\cdot10\,Hz=62.8\,rad\,s^{-1} \quad ; \quad x=A\cos(\omega t) \quad ; \quad v=-A\,\omega\sin(\omega t)$$

$$v_{eq}=v_{max}=A\,\omega=0.2\,m\cdot62.8\,rad\,s^{-1}=12.6\,m\,s^{-1}$$

$$k=\omega^2 m=(62.8\,rad\,s^{-1})^2\cdot2.72\cdot10^5 kg=1.07\cdot10^9 N\,m^{-1}$$

$$E=V+K=\frac{1}{2}k\,A^2=\frac{1}{2}1.07\cdot10^9 N\,m^{-1}\cdot(0.2\,m)^2=2.14\cdot10^7 J$$

7. A damped oscillator is made of a block oscillating at the end of a spring. Suppose that the mass of the block is $m=250\,g$ **, the spring constant is** $k=85\,N\,m^{-1}$ **and the damping constant of the spring is** $b=0.14\,s^{-1}$ **. What is the period of the motion? How long does it take for the amplitude of the damped oscillations to drop to half of its initial value?**

$m=250\,g=$
$=0.25\,kg$

$k=85\,N\,m^{-1}$

$b=0.14\,s^{-1}$

$T=?$

$x_{max}(t_1)=$
$=\frac{1}{2}x_{max}(0)$

$t_1=?$

$$x(t)=A\,e^{-bt}\cos(\omega' t+\varphi) \quad ; \quad \omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{85\,N\,m^{-1}}{0.25\,kg}}=18.4\,rad\,s^{-1}$$

$$\omega'=\sqrt{\omega^2-b^2}=\sqrt{(18.4\,rad\,s^{-1})^2-(0.14\,s^{-1})^2}=18.4\,rad\,s^{-1}$$

$$\omega'=\frac{2\pi}{T} \quad ; \quad T=\frac{2\pi}{\omega'}=\frac{2\pi}{18.4\,rad\,s^{-1}}=0.341\,s$$

$$x_{max}(0)=A \quad ; \quad x_{max}(t_1)=A\,e^{-bt_1} \quad ; \quad x_{max}(t_1)=\frac{1}{2}x_{max}(0) \quad ; \quad A\,e^{-bt_1}=\frac{A}{2}$$

$$t_1=\frac{\ln2}{b}=\frac{\ln2}{0.14\,s^{-1}}=4.95\,s$$

8. The amplitude of a lightly damped oscillator decreases by 3% **during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?**

$A(t+T)=$
$=\frac{97}{100}A(t)$

$\frac{E(t+T)}{E(t)}=?$

$$E=V+K=\frac{1}{2}k\,A^2$$

$$\frac{E(t+T)}{E(t)}=\frac{\frac{1}{2}k\,A(t+T)^2}{\frac{1}{2}k\,A(t)^2}=\frac{\frac{1}{2}k\left[\frac{97}{100}A(t)\right]^2}{\frac{1}{2}k\,A(t)^2}=\left(\frac{97}{100}\right)^2=0.941=94.1\%$$

9. An external harmonic force is acting on a damped steel string. The frequency of the acting force is Ω and the string displacement due to oscillations is described by the formula $x(t)=A\sin(\Omega t+\varphi)$ **. The amplitude of the forced oscillations is given by** $A=\frac{C}{\sqrt{(\omega^2-\Omega^2)^2+4b^2\Omega^2}}$ **, where** $\omega=50\,rad\,s^{-1}$ **is the natural frequency of the string,** $b=0.5\,s^{-1}$ **is the damping constant and** C **is a real constant. Find the frequency of the acting force in resonance.**

$\omega=50\,rad\,s^{-1}$

$b=0.5\,s^{-1}$

$\Omega_r=?$

$$\Omega_r=\sqrt{\omega^2-2b^2}=\sqrt{(50\,rad\,s^{-1})^2-2(0.5\,s^{-1})^2}=50\,rad\,s^{-1}$$