Seminary exercise Nr. 8
Fluid mechanics

1. What fraction of the volume of an iceberg (density $917\,kg\,m^{-3}$) would be visible if the iceberg floats in the ocean (salty water, density $1024\,kg\,m^{-3}$) and in a river (fresh water, density $1000\,kg\,m^{-3}$)?

$\rho_{ice}=917\,kg\,m^{-3}$

$\rho_{sw}=1024\,kg\,m^{-3}$

$\rho_{fw}=1000\,kg\,m^{-3}$

$\frac{V_{sw}^{out}}{V}=?$

$\frac{V_{fw}^{out}}{V}=?$

$$F_p=mg=\rho_{ice}V\,g \quad ; \quad F_A=\rho_w V_w^{in}\,g=\rho_w\left(V-V_w^{out}\right)g \quad ; \quad V_w^{in}+V_w^{out}=V$$

At equilibrium: $F_p=F_A \quad ; \quad \rho_{ice}V\,g=\rho_w\left(V-V_w^{out}\right)g \quad ; \quad \rho_{ice}V=\rho_w V-\rho_w V_w^{out}$

$$V\left(\rho_{ice}-\rho_w\right)=-\rho_w V_w^{out} \quad ; \quad \frac{V_w^{out}}{V}=\frac{\rho_w-\rho_{ice}}{\rho_w}$$

$$\frac{V_{sw}^{out}}{V}=\frac{\rho_{sw}-\rho_{ice}}{\rho_{sw}}=\frac{1024\,kg\,m^{-3}-917\,kg\,m^{-3}}{1024\,kg\,m^{-3}}=0.1045$$

$$\frac{V_{sw}^{out}}{V}=\frac{\rho_{fw}-\rho_{ice}}{\rho_{fw}}=\frac{1000\,kg\,m^{-3}-917\,kg\,m^{-3}}{1000\,kg\,m^{-3}}=0.083$$

3. At a depth of $10.9\,km$, the Challenger Deep in the Marianas Trench of the Pacific Ocean is the deepest site in any ocean. Yet, in 1960, Donald Walsh and Jacques Piccard reached the Challenger Deep in the bathyscaph Trieste. Assuming that seawater has a uniform density of $1024\,kg\,m^{-3}$, calculate the hydrostatic pressure that the Trieste had to withstand.

$d=10.9\,km=$
$=1.09\cdot10^4\,m$

$\rho=1024\,kg\,m^{-3}$

$p=?$

$$p=\rho\,d\,g=1024\,kg\,m^{-3}\cdot1.09\cdot10^4\,m\cdot9.81\,m\,s^{-2}=1.09\cdot10^8\,Pa$$

8. The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. Calculate the average speed of the blood in the aorta if the flow rate is $5\,L\,min^{-1}$. The aorta has a radius of $10\,mm$. The speed of blood in the capillaries is about $0.33\,mm\,s^{-1}$. The average diameter of a capillary is $8\,\mu m$. Calculate the approximate number of capillaries in the blood circulatory system.

$f_a=5\,L\,min^{-1}=$
$=8.33\cdot10^{-5}\,m^3 s^{-1}$

$r_a=10\,mm=$
$=0.01\,m$

$v_c=0.33\,mm\,s^{-1}=$
$=3.3\cdot10^{-4}\,m\,s^{-1}$

$d_c=8\,\mu m=$
$=8\cdot10^{-6}\,m$

$v_a=?$

$n_c=?$

$$v_a=\frac{f_a}{A_a}=\frac{f_a}{\pi r_a^2}=\frac{8.33\cdot10^{-5}\,m^3 s^{-1}}{\pi(0.01\,m)^2}=0.265\,m\,s^{-1}$$

$$A_a v_a=n_c A_c v_c \quad ; \quad \pi r_a^2 v_a=n_c\,\pi\left(\frac{d_c}{2}\right)^2 v_c$$

$$n_c=\frac{4\,r_a^2 v_a}{d_c^2 v_c}=\frac{4\cdot(0.01\,m)^2\cdot0.265\,m\,s^{-1}}{(8\cdot10^{-6}\,m)^2}\cdot3.3\cdot10^{-4}\,m\,s^{-1}=40150$$

9. Ethanol of density $\rho=791\,kg\,m^{-3}$ flows smoothly through a horizontal pipe which changes cross-sectional area from $A_1=1.23\cdot10^{-3}\,m^2$ to $A_2=A_1/2$. The pressure difference between the wide and narrow sections of the pipe is $4120\,Pa$. What is the volume flow rate of ethanol?

$\rho=791\,kg\,m^{-3}$

$A_1=1.23\cdot10^{-3}\,m^2$

$A_2=\dfrac{A_1}{2}$

$\Delta p=4120\,Pa$

$f=?$

$$\frac{p_1}{\rho}+\frac{v_1^2}{2}=\frac{p_2}{\rho}+\frac{v_2^2}{2} \quad ; \quad f=A_1 v_1=A_2 v_2 \quad ; \quad v_1=\frac{A_2}{A_1}v_2=\frac{\frac{A_1}{2}}{A_1}v_2=\frac{v_2}{2}$$

$$\frac{p_1}{\rho}+\frac{\frac{v_2^2}{4}}{2}=\frac{p_2}{\rho}+\frac{v_2^2}{2} \quad ; \quad \frac{p_2-p_1}{\rho}=\frac{v_2^2}{2}\left(\frac{1}{4}-1\right) \quad ; \quad \frac{\Delta p}{\rho}=\frac{3}{8}v_2^2$$

$$v_2=\sqrt{\frac{8\Delta p}{3\rho}}=\sqrt{\frac{8\cdot4120\,Pa}{3\cdot791\,kg\,m^{-3}}}=3.73\,m\,s^{-1}$$

$$f=A_2 v_2=\frac{A_1}{2}v_2=\frac{1.23\cdot10^{-3}\,m^2}{2}\cdot3.73\,m\,s^{-1}=2.29\cdot10^{-3}\,m^3 s^{-1}$$

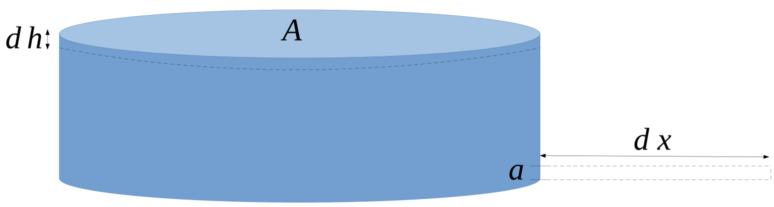
12. A cylindrical tank with a diameter $D=2\,m$ is filled with water to a height $H=40\,cm$. A hole of cross-sectional area $a=6.5\,cm^2$ in the bottom of the tank allows water to drain out. How long does it take to drain the whole tank?

$D=2\,m$

$H=40\,cm=$
$=0.4\,m$

$a=6.5\,cm^2=$
$=6.5\cdot10^{-4}\,m^2$

$t=?$



$$A\,dh=a\,dx \quad ; \quad dx=\frac{A}{a}dh \quad ; \quad \sqrt{2gh}=v=\frac{dx}{dt}=\frac{A}{a}\frac{dh}{dt}$$

$$\sqrt{2gh}=\frac{A}{a}\frac{dh}{dt} \quad ; \quad \frac{a\sqrt{2g}}{A}dt=\frac{dh}{\sqrt{h}} \quad ; \quad \int_0^t \frac{a\sqrt{2g}}{A}dt'=\int_0^H \frac{dh}{\sqrt{h}}$$

$$\left[\frac{a\sqrt{2g}}{A}t'\right]_0^t=\left[2\sqrt{h}\right]_0^H \quad ; \quad \frac{a\sqrt{2g}}{A}t=2\sqrt{H}$$

$$t=\frac{2A\sqrt{H}}{a\sqrt{2g}}=\frac{2\pi\left(\frac{D}{2}\right)^2\sqrt{H}}{a\sqrt{2g}}=\frac{2\pi\left(\frac{2\,m}{2}\right)^2\sqrt{0.4\,m}}{6.5\cdot10^{-4}\,m^2\sqrt{2\cdot9.81\,m\,s^{-2}}}=1380\,s$$