From Fourier Series to Analysis of Non-stationary Signals – VII

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Contents

Short Time Fourier Transform

An overview of Fourier transforms

	continuous in time	discrete in time periodic in frequenc
continuous in frequency	$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$	$x(n) = \frac{T}{2\pi} \int_{-\pi/T}^{+\pi/T} X(e^{j\omega T}) e$
	$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$	$X(e^{j\omega T}) = \sum_{n=-\infty}^{\infty} x(n) e^{-}$
	Fourier transform	Fourier transform $t =$
y		

What is wrong with Fourier Transform?

Let's consider basis functions: $\sin(\omega t)$, $\cos(\omega t)$ and $\delta(t)$

support region	in time	in frequency
$\sin(\omega t)$	$(-\infty, \infty)$	0
$\cos(\omega t)$	$(-\infty, \infty)$	0
$\delta(t)$	0	$(-\infty, \infty)$

- The basis functions $\sin(\omega t)$ and $\cos(\omega t)$ are not localized in time!
- The $\delta(t)$ is not localized in frequency!

We have learned to localize a signal in time domain by windowing
$\Rightarrow$ short time Fourier transform (STFT)

Short Time Fourier Transform

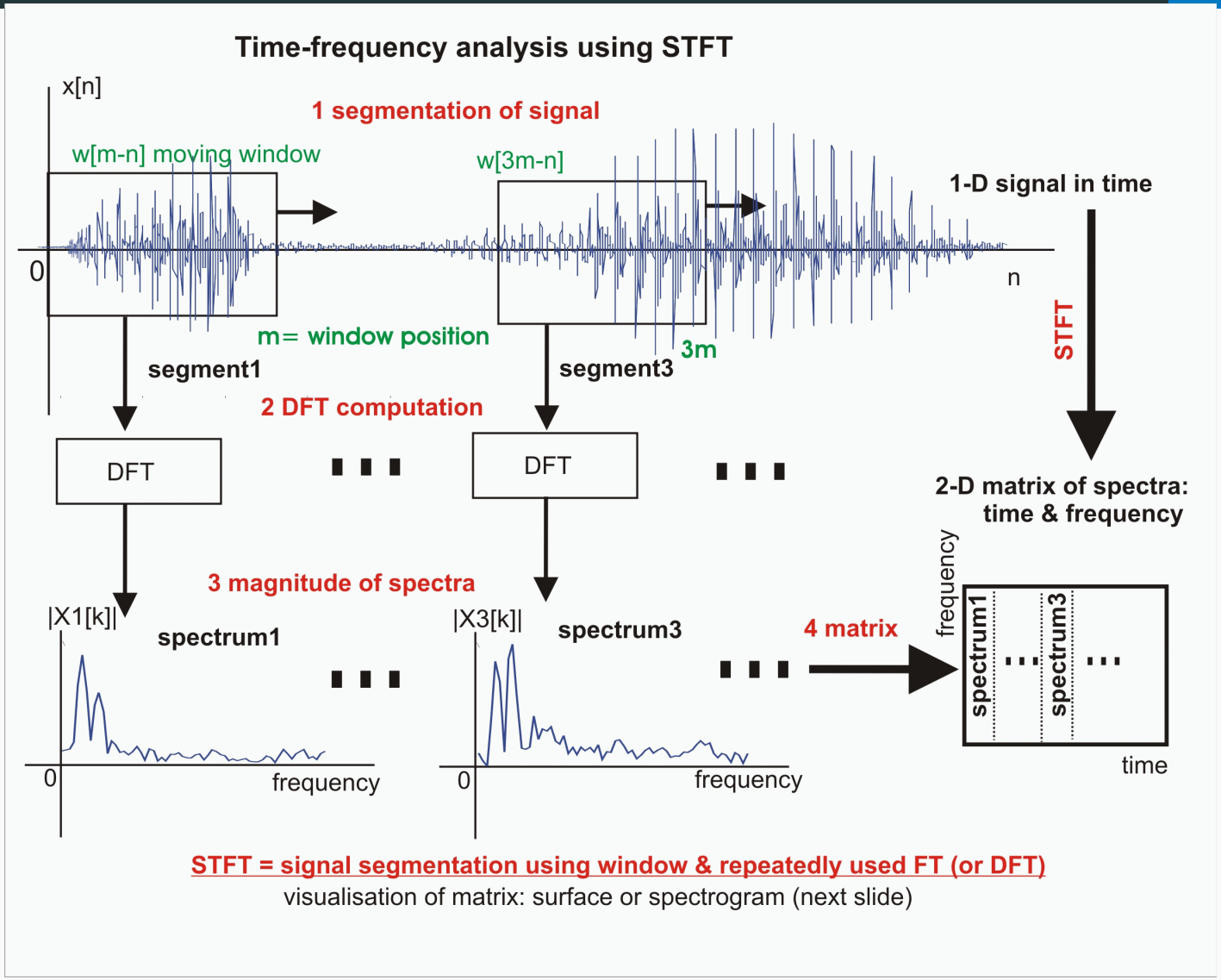
- When sampling an audio signal at a sampling rate 44.1 kHz
- ... 1 hour of stereophonic music would be $44100 \times 2 \times 60 \times 60 = 317520000$ samples !
- If we want to compute DFT, the closest power-of-two FFT is $2^{28} = 268435456$ per channel.
- A better approach is to break the long signal into small segments and analyze each one with FFT $\Rightarrow$ short time Fourier transform

Short Time Fourier Transform

- Assume that $x[m]$ is an infinitely long sequence
- in order to localize energy in time and frequency we segment the signal into short-time pieces and calculate DFT of each one
- sampled STFT for a window defined in the region $0 \leq m \leq M-1$ is given by

$$X[k, \ell L] = \sum_{m=0}^{M-1} x[\ell L - m] w[m] \exp\left(-\mathrm{j}2\pi \frac{km}{N}\right)$$

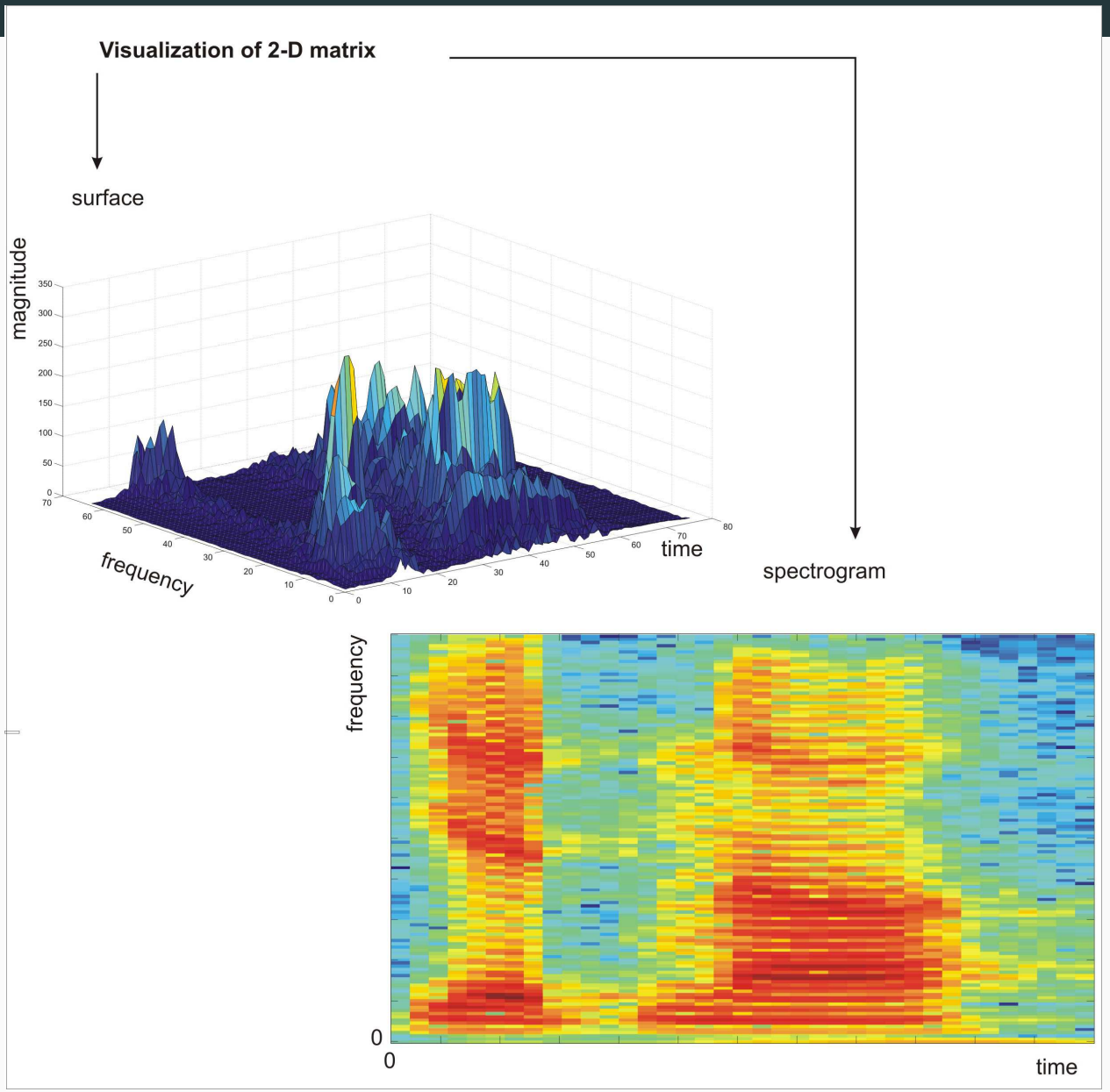
Short Time Fourier Transform



STFT = signal segmentation using window & repeatedly used FT (or DFT)

visualisation of matrix: surface or spectrogram (next slide)

Short Time Fourier Transform



Spectrograms

Spectrograms

In MATLAB the command

```
spectrogram(x,window,noverlap,nfft,fs,'yaxis')
```

performs short-time Fourier transform and plots a 2D frequency-time diagram, where

- `x` is the signal specified by vector **x**.
- if `window` is an integer, `x` is divided into segments of length equal to that integer value
- otherwise, `window` is a Hamming window of length `nfft`
- `noverlap` is the number of samples each segment of `x` overlaps

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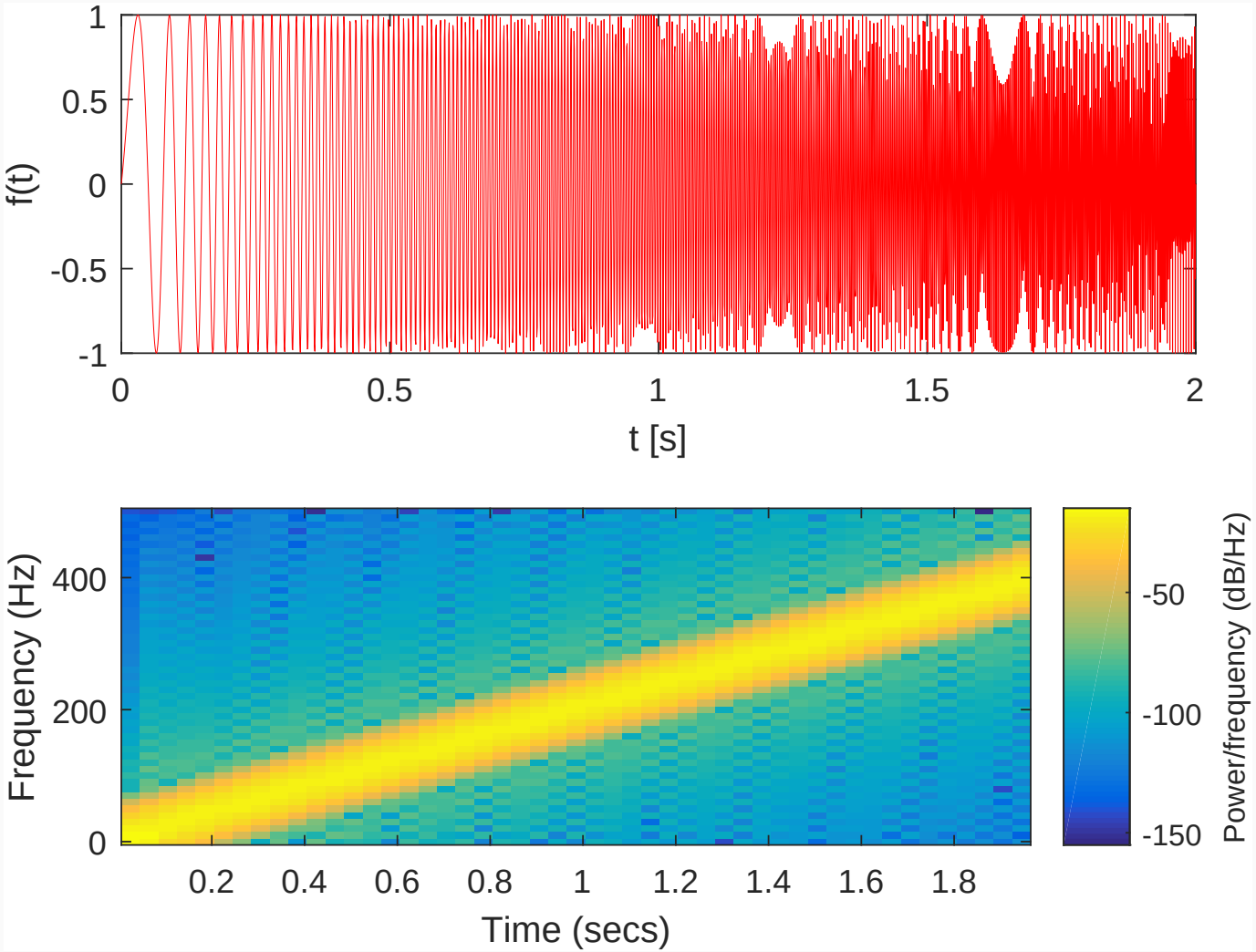
Spectrograms

- `nfft` is the FFT length and is the maximum of 256 or the next power of 2 greater than the length of each segment of `x`
- `fs` is the sampling frequency, which defaults to normalized frequency
- using `'yaxis'` displays frequency on the y-axis and time on the x-axis

We also use command `colorbar` which appends a color scale to the current axes.

Chirp signal analysis $\sin(2\pi(f_0 + \alpha t)t)$



Matlab project

MATLAB project – Spectrogram I

a) Start MATLAB. Load in the audio signal with commands

```
filename = 'flute-C4.wav';
[x1 sr1] = audioread(filename);
```

b) The sampling rate is 11 025 Hz, and the signal contains 36 250 samples.

Q: If we consider this signal as sampled on an interval $[0, T)$, what is the time duration of the flute sound ?

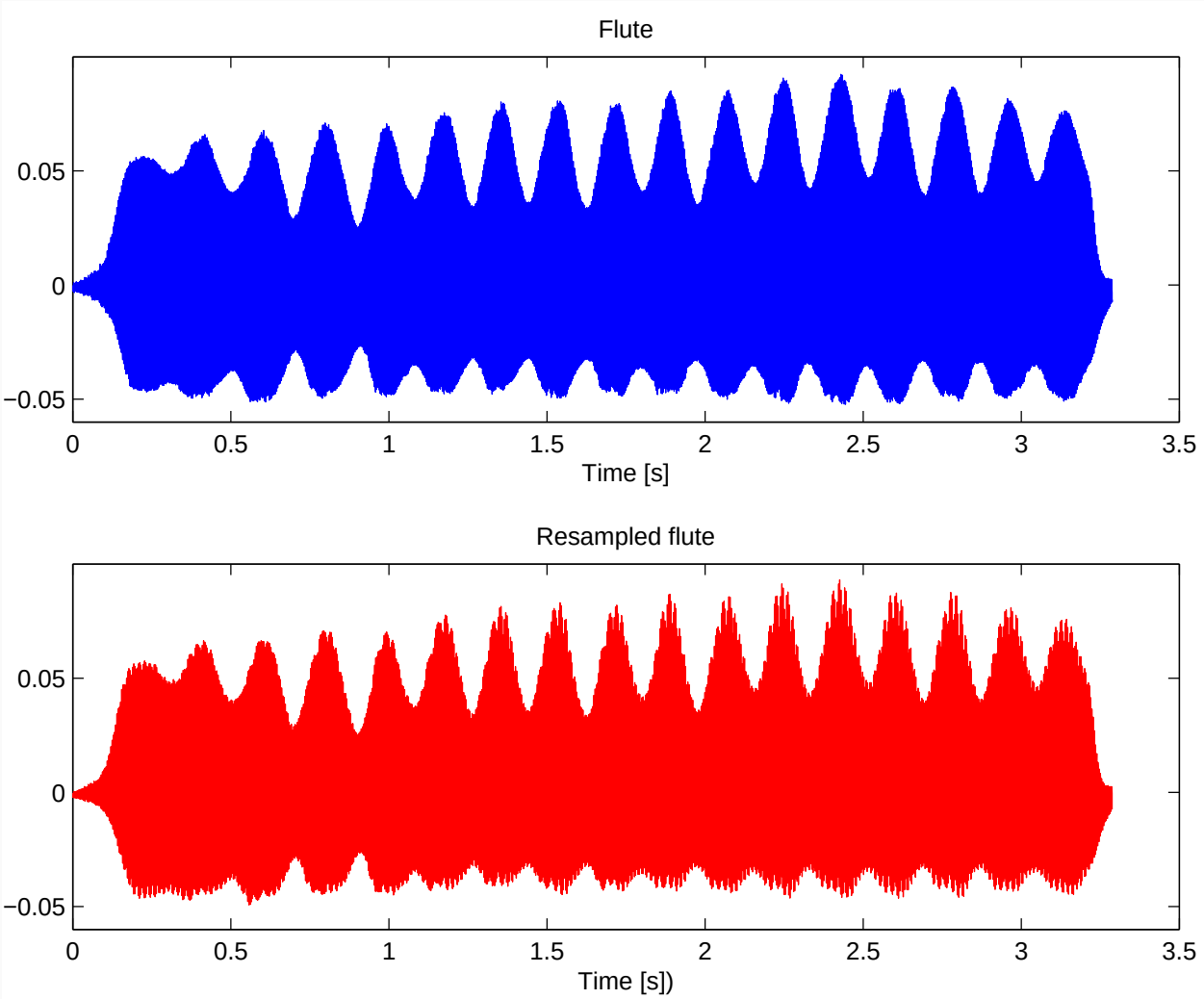
c) Use command `soundsc(x1,sr1)` to obtain flute sound

click to play

d) Resample the audio signal by $f_r = 4000$ Hz and write the sound file to disk using

```
audiowrite('flute-resampled.wav',x2, sr2);
```

MATLAB project – Spectrogram I



e) Compute the DFT of the signal with `X1 = fft(x1(1:1024));` and `X2 = fft(x2(1:1024));`

f) DFT of real-valued signals is always symmetric around `fr/2` so we only need to plot the first half. Display the magnitude of the Fourier transform using

```
plot(f1(1:end/2+1), abs(X1(1:end/2+1)))}
```

g) **Q:** What is the approximate fundamental frequency of the flute note C4?

MATLAB project – Spectrogram I

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g) **Q:** What is the approximate fundamental frequency of the flute note C4? A: Find the bin corresponding to the first peak in the magnitude spectrum.

MATLAB project Spectrogram I

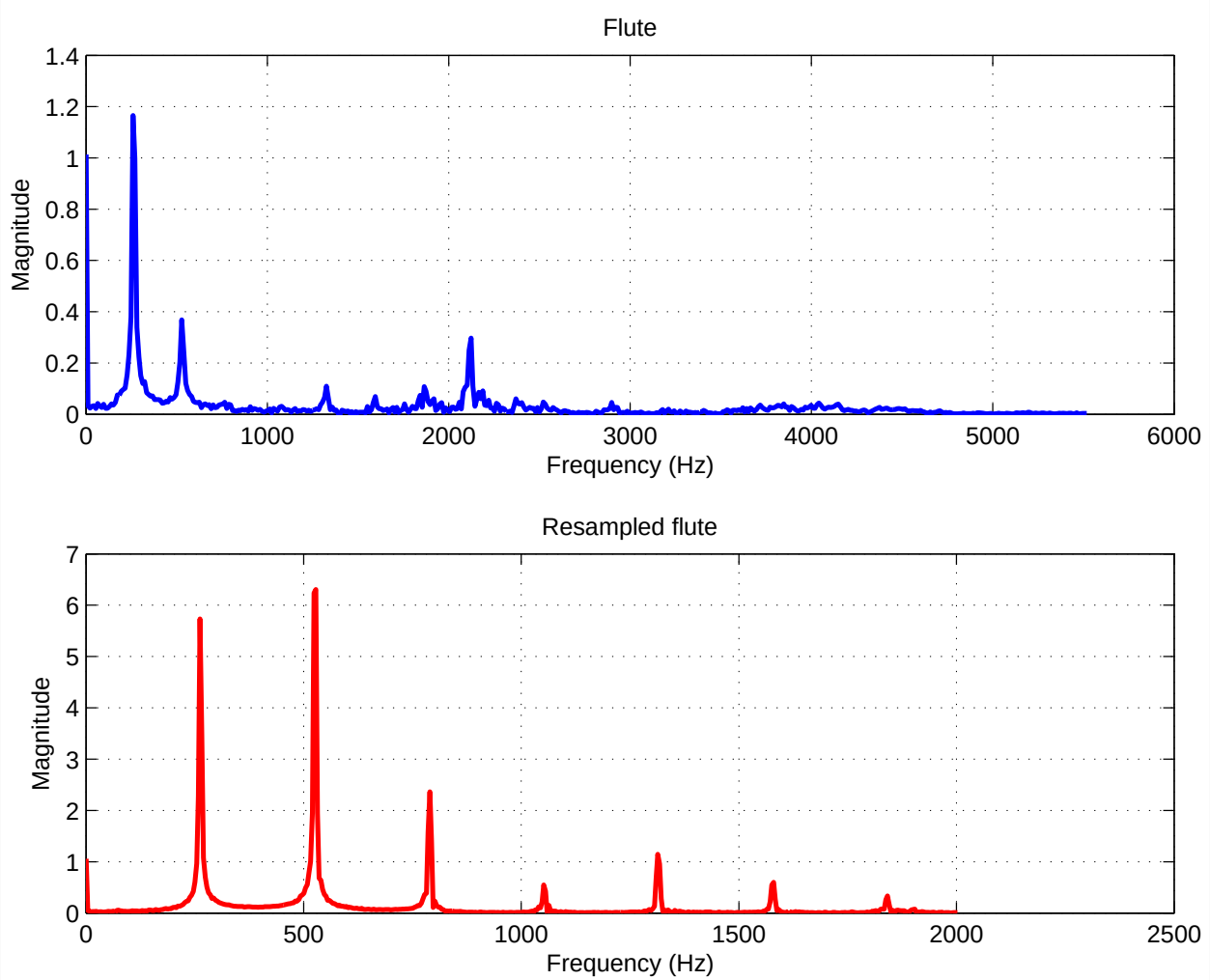
e) You can use a systematic way to find the frequency of the peaks in spectrum `abs(X2)` using following commands:

```
% find local maxima
mag = abs(X2);
mag = mag(1:end/2+1);
peaks = (mag(1:end-2) < mag(2:end-1)) &
(mag(2:end-1) > mag(3:end));
```

f) Then evaluate the peaks at corresponding frequencies above a threshold:

```
peaks = peaks & mag(2:end-1) > 0.5;
fmax = f2(peaks)
```

MATLAB project Spectrogram I

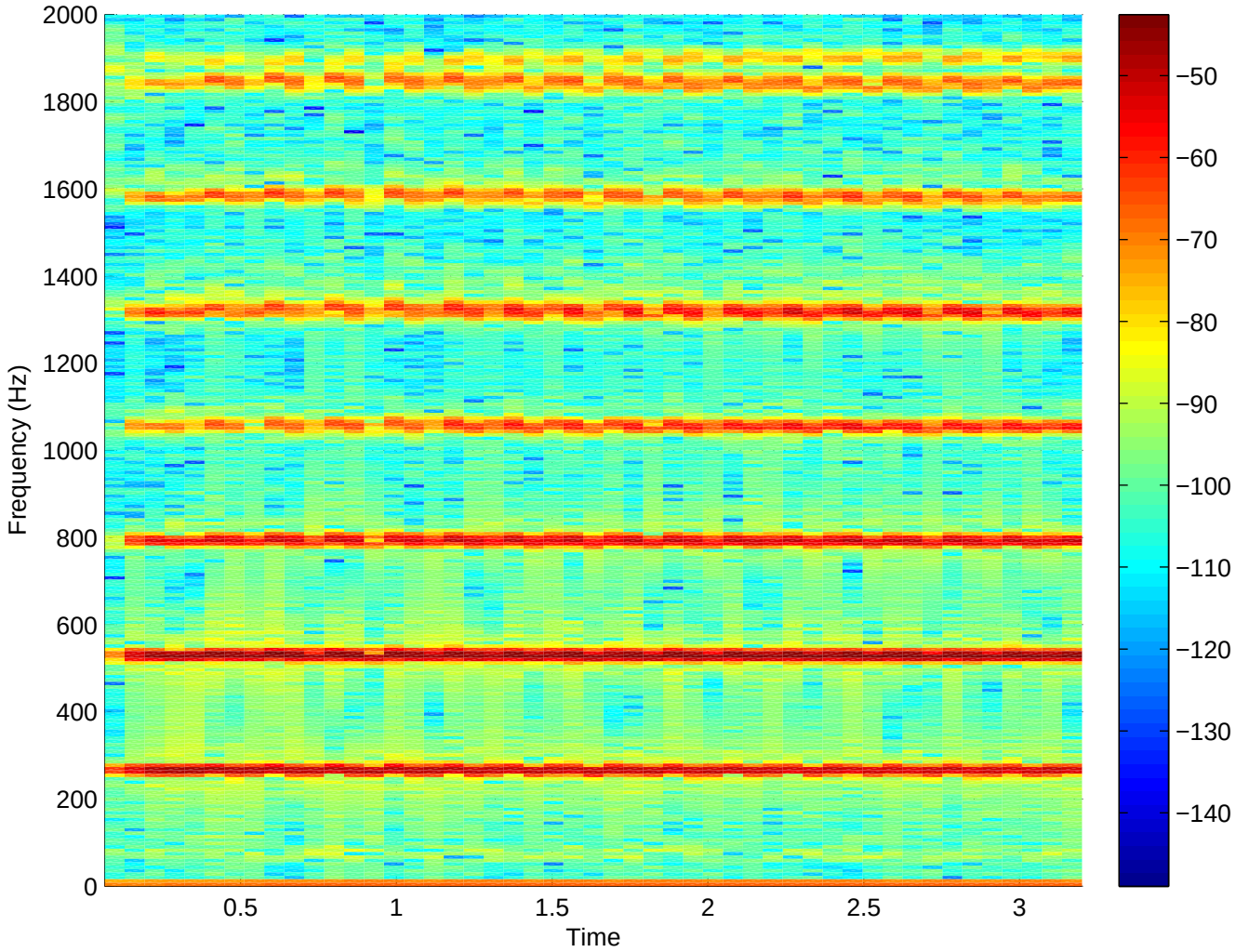


g) Finally we will use Spectrogram with following specifications:

```
nwin = 512; % samples of a window
noverlap = 256; % samples of overlaps
nfft = 512; % samples of fast Fourier
transform
f = figure(1)
spectrogram(x1, nwin, noverlap, nfft, sr2,
'yaxis');
colorbar % not stricly necessary
print -djpeg90 figure-spect % or ...
saveas(f, 'figure-spect.eps', 'epsc);
```

h) Carefully study the options for the spectrogram using help!

MATLAB project Spectrogram – I



MATLAB project Spectrogram – II

a) Construct a signal `f` of two sinusoids on interval $t \in [0, 2)$ sampled at 1 kHz

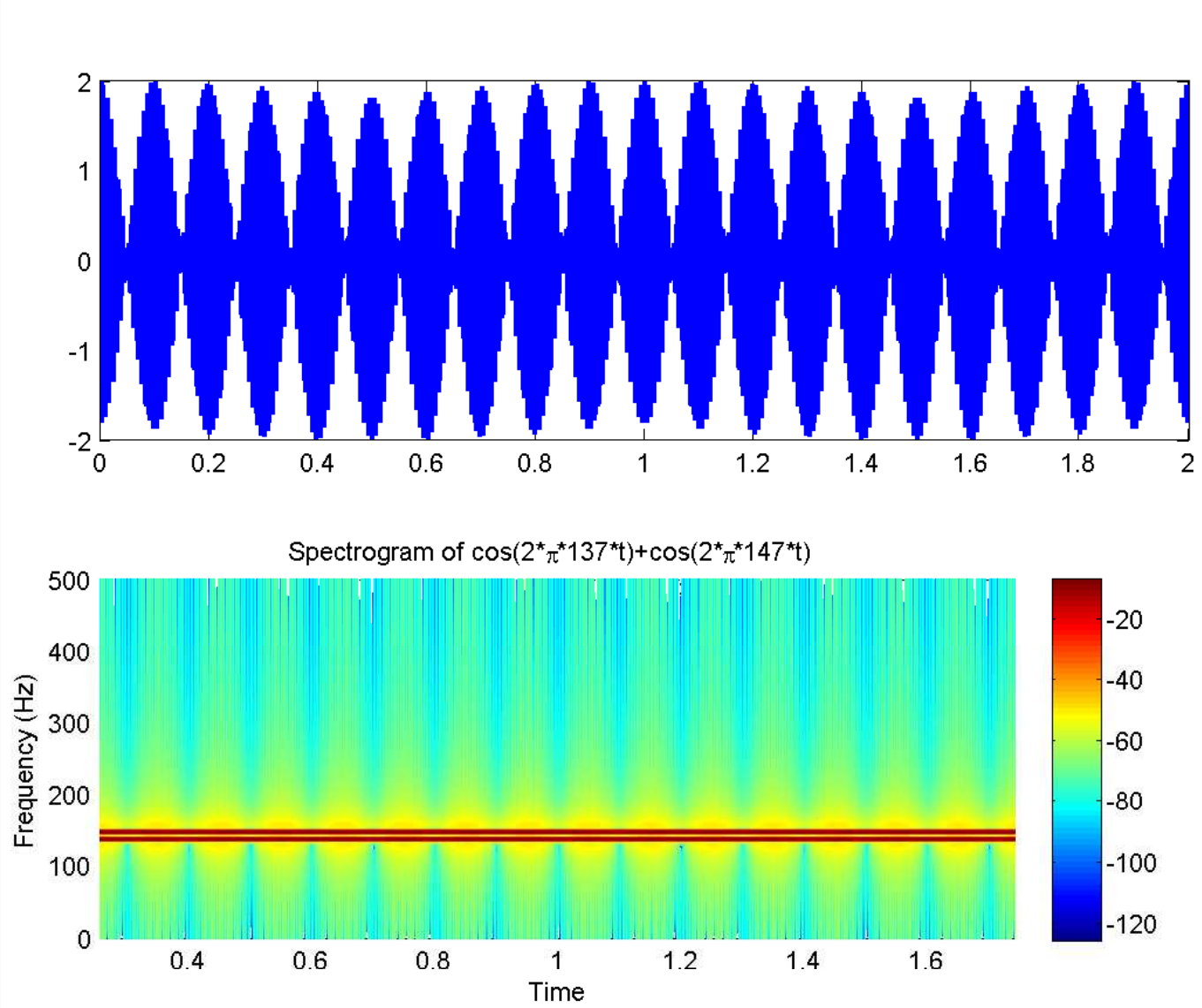
```
t=[0:1999]/1000;
f=cos(2*pi*137*t)+cos(2*pi*147*t);
```

b) Compute the DFT of `f` and display its magnitude in time and frequency domain.

c) Construct a spectrogram using

```
spectrogram(f,512,510,512,1000,'yaxis');
```

MATLAB project – Spectrogram II



MATLAB project Spectrogram II